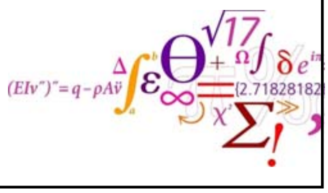
TopWing
Ultra-high resolution structural optimization

Villum fonden - The NextTop project
PRACE: CURIE Thin Nodes (TN) (GENCI@CEA, France)

Associate Professor Niels Aage

& Ole Sigmund, Erik Andreassen & Boyan Lazarov.



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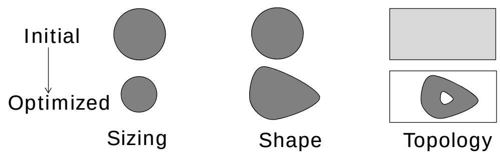
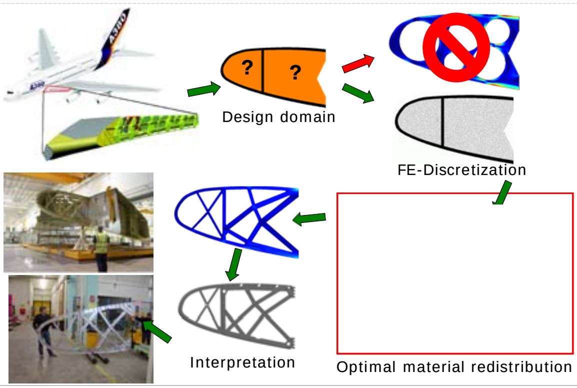
Classes of structural optimization

Classes of structural optimization methods:



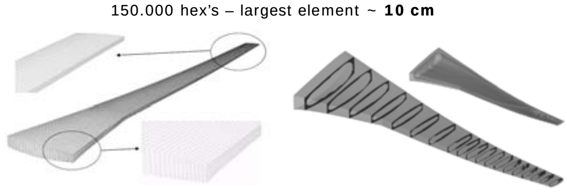
Topology Optimization in Aerospace

Bendsøe and Kikuchi (1988)

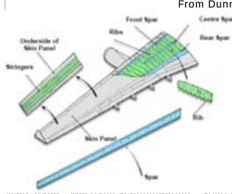


Topology Optimization in Aerospace

150.000 hex's – largest element ~ **10 cm**



From Dunning et.al., AIAA, 2014.

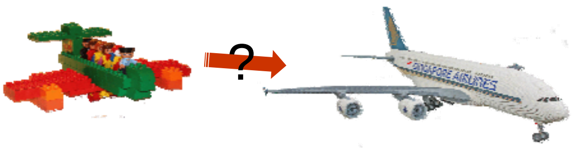


"the resulting structure typically bears ***no resemblance to traditional rib/spar networks****, which may indicate one of two things. The first is that the* ***appropriate physics, load cases****, and/or constraint boundaries were not included in the optimization problem, and if they had been, the resulting topology would qualitatively approach a lattice of ribs and spars. The second is that* ***the design problem was properly defined****, and that the non-traditional topology may present an interesting new direction for efficient wing structures".*

Topology Optimization in Aerospace

Ultra high resolution TopOpt

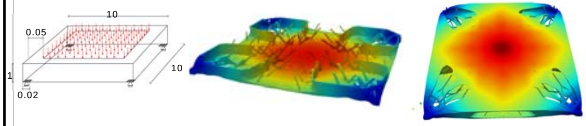
(overcoming the Duplo problem)



The motivation

Why ? Use our PETSc based TopOpt framework:

Topology optimization using PETSc: An easy-to-use, fully parallel, open-source topology optimization framework, Aage, N; Andreassen, E. & Lazarov, B.S., SMO, 51:565-572, **2015.**



How ? PRACE application for CURIE super computer (80.000 cores)

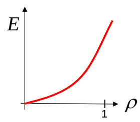
Design problem formulation

Minimum compliance design problem:

$$
\begin{aligned}
\min_{\rho \in \mathbb{R}^n} \quad & \phi(\boldsymbol{u},\boldsymbol{\rho}) = \textstyle\sum_i \alpha_i \boldsymbol{F}_i^T \boldsymbol{u}_i \\
\text{s.t.} \quad & \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u}_i = \boldsymbol{F}_i, \quad i = 1,\ldots,m \\
& V(\boldsymbol{\rho})/V^* - 1 \le 0 \\
& 0 < \rho^{\min} \le \rho_j \le 1 \,, j = 1,\ldots,n
\end{aligned}
$$

Design interpolation:

- SIMP method (Solid Isotropic Material with Penalization)
- Ensures solid/void designs



$$\boxed{E(\rho_e) = \rho_e^p E_0}$$

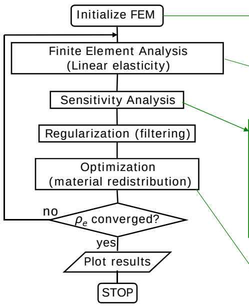
$$p > 1$$

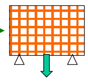
Predominant solution methods

Solving PDE constrained optimization problems ?

- Simultaneous ANalysis and Design (SAND)
  - Solve for state and design variables.
  - Feasibility issue.
  - Many constraints.
- Nested formulation (engineering approach)
  - Consider the state variables to be an implicit function of the design variable.
  - Design is always physically sound.

The Topology Optimization Process





$KU = F$ or $R(U) = 0$

Sensitivity analysis by adjoint method

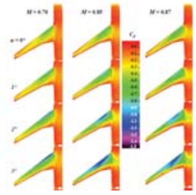
$$\frac{d\Phi}{d\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \lambda^T \left(\frac{\partial K}{\partial \rho_e} U + \frac{\partial F}{\partial \rho_e} \right)$$

$$K^T \lambda = -\frac{\partial \Phi}{\partial U}$$

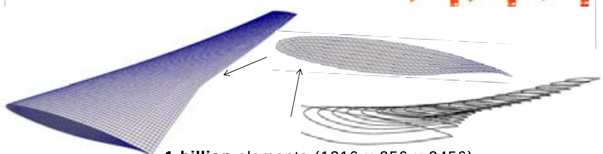
Mathematical Programming,
Optimality Criteria (OC) and/or the
Method of Moving Asymptotes (MMA)

NASA Common Research Model

Geometry and pressure load data from NASA:



Meshing by structured slices:



~**1 billion** elements (1216 x 256 x 3456)…
… largest element side **0.8 cm** (wing is ~ 28m x 12m x 2m)

Where we spend the time ?

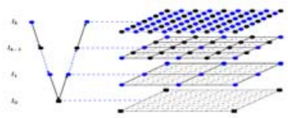
The **state problem** is the major limiting factor:
- why solving a linear Poisson type problem is so difficult?

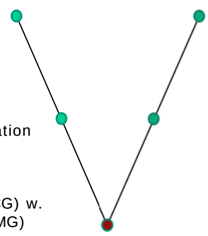
- Stiffness contrast – 6 to 9 orders of magnitude.
- Aspect ratio of wing structure.
- Length scale.
- The pressure loads, i.e. spatial content.

Solver (part I)

The linear elasticity problem:

- Outer Krylov solver: **Flexible** GMRES ($\mathcal{K}_n = \mathrm{span}\{r, Ar, A^2r, ..., A^{n-1}r\}$)
- Preconditioner: **Geometric** Multi Grid (GMG)





- **Smoother**: Chebyshev w. Successive Over Relaxation (SOR)
- **Coarse solver**: Conjugate Gradients (CG) w. Algebraic Multi Grid (AMG)

Motivation for chosen solver

Criterias for solver construction:

- Must be able to solve the optimization problem in 1 million CPU hours.
- Must be highly scalable:
  - Both numerical and parallel
- More involved preconditioners (our implementation/test) are slower:
  - ASM
  - Multi-level/scale
  - Full AMG
  - FETI/BDDC

Solver (part II)

Solving the optimization problem:

- Optimality condition – for the nested formulation:

$$L(\boldsymbol{\rho}, \lambda, \boldsymbol{\mu}_{\min}, \boldsymbol{\mu}_{\max}) = \phi(\boldsymbol{u}, \boldsymbol{\rho}) + \lambda(V(\boldsymbol{\rho})/V^* - 1) + \boldsymbol{\mu}_{\min}(\boldsymbol{0} - \boldsymbol{\rho}) + \boldsymbol{\mu}_{\max}^T(\boldsymbol{\rho} - \boldsymbol{1})$$

- Which leads to the 1st order condition (unbound variables):

$$-\frac{\frac{\partial \phi}{\partial \rho_e}}{\lambda \frac{\partial g}{\partial \rho_e}} = 1, \quad \text{for } e = 1, \ldots, n$$

- Failure of the commonly used Convex Seperable Approximation approach, e.g. MMA is much slower than an OC above 100 million dofs.

Visualization and post processing

Storage and machines from KU (acknowledge!)
- The mesh, state field and design fields ~100GB for one iteration.
- Thanks to Troels Haugbølle and Åke Norlund from KU (Niels Bohr Institute)
- Visualization performed on a 32-core machine with 800GB memory.

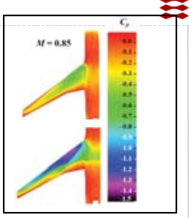
Time spend on visualizing:
- Much more than running the optimization!

Load cases and performance

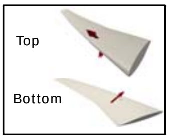
Load cases:
- 0-degree incidents at cruise speed.
- 4-degree incidents at cruise speed.
- Engine load

Number of processes used for full wing model:
- 8000 cores (tested on 30.000)

Time for optimization cycle:
- 400 – 1200 PDE solves
  - State solver timing: from ~4s to ~600s
- Min / max number of Krylov cycles
  - 21 / 135
- Total time from 1-5 days.



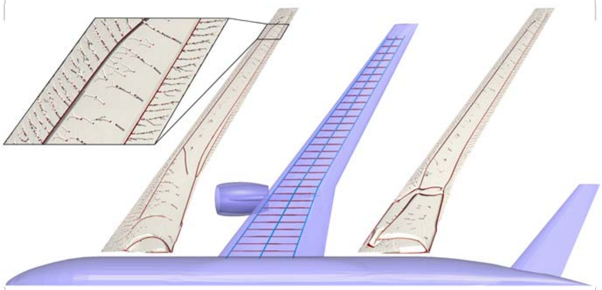
Aerodynamic loads



Engine load

Results: 650 million elements

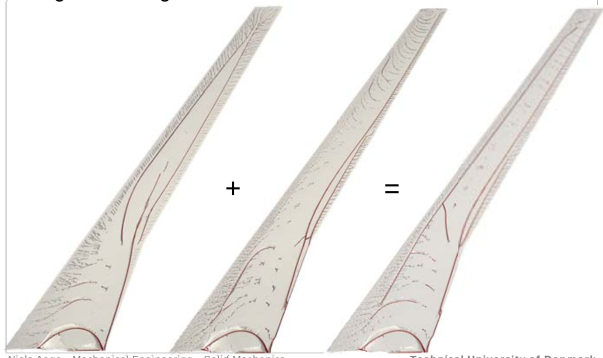
Wing box design – comparison to conventional layout



Niels Aage, Mechanical Engineering, Solid Mechanics — Technical University of Denmark

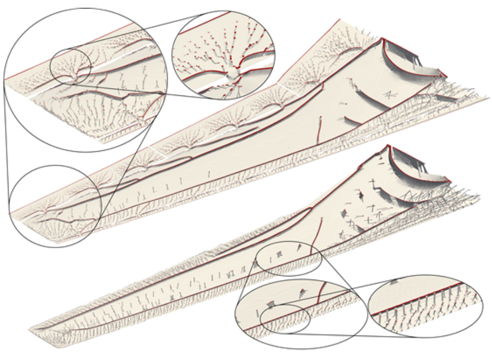
Results: 650 million elements

Wing box design – Influence of loadcases



Niels Aage, Mechanical Engineering, Solid Mechanics — Technical University of Denmark

Results: 1075 million elements



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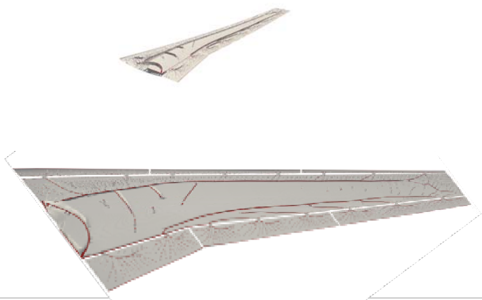

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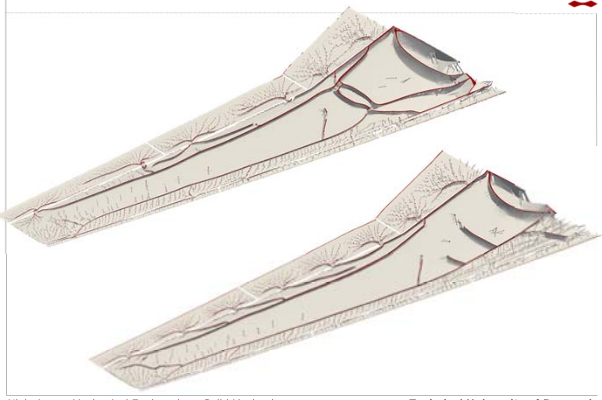

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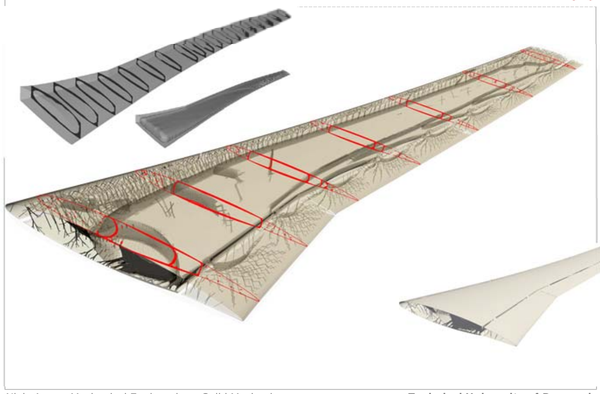
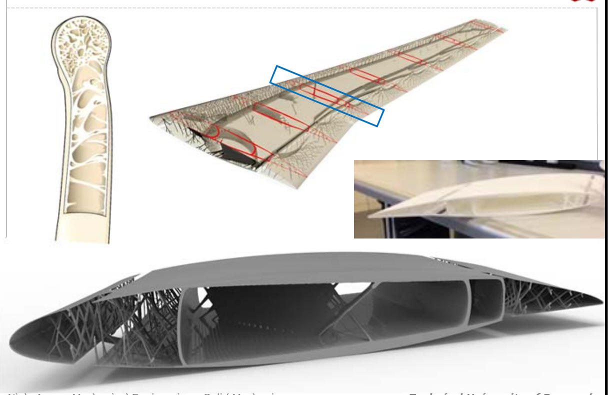


Results: Mega scale 3D printing



Summary of study

What have we accomplished:

- Unprecedented level of details for structural optimization.
- Increased the resolution by 2 orders of magnitude.
- Shown that rib and spar structures are indeed present in optimized wing structures.
- Demonstrated the potential of ultra-high resolution TopOpt on a engineering structural design problem.

What have we not considered:

- Non-linear effects.
- Buckling.
- Anisotropy.
- Dynamics.
- Aero-elasticity.

Perspectives

Where/when can the presented work be used?
- In academia: Use results to train aerospace students.
- In industry: not likely today, nor tomorrow – but in a few decades.

How we envision the presented work can be used in industry:
1. Optimize using ultra-high resolution.
2. Extract geometry and remesh (<50 million elements).
3. Check with standards (>100 load situations).
4. Add possible loadings scenarios – if failed – and goto 1.

In the near future:
- Multi-physics structural design problems.

In the (hopefully not so) far future
- 3D printing of full wing structure☺

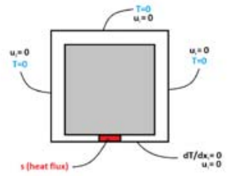
Natural convection problems

- Steady-state incompressible Navier-Stokes equations coupled to the convection-diffusion equation through the Bousinessq approximation:

$$u_j \frac{\partial u_i}{\partial x_j} - Pr \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial p}{\partial x_i} = -\alpha(\mathbf{x}) u_i - Gr Pr^2 e_i^g T$$

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$u_j \frac{\partial T}{\partial x_j} - \frac{\partial}{\partial x_j}\left(K(\mathbf{x}) \frac{\partial T}{\partial x_j}\right) = s(\mathbf{x})$$



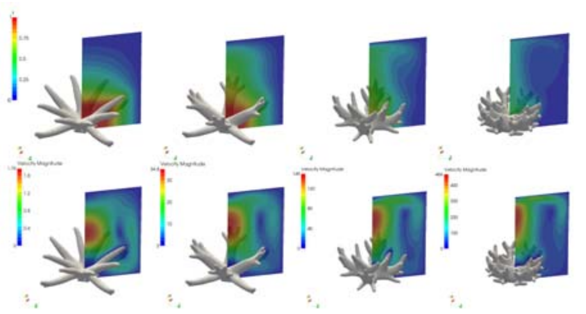
- $\alpha(\mathbf{x})$: Brinkman penalisation

$$\alpha(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \in \Omega_f \\ \infty & \text{if } \mathbf{x} \in \Omega_s \end{cases}$$

- $K(\mathbf{x})$: Effective conductivity

$$K(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_f \\ C_k{}^{-1} = \frac{k_s}{k_f} & \text{if } \mathbf{x} \in \Omega_s \end{cases}$$

Natural convection – cavity problem



Increasing Grashof number from left to right - $Gr \in \{10^3, 10^4, 10^5, 10^6\}$

Alexandersen, Aage, Sigmund, **2015**, in prep.

Coupled multiphysics: 3D

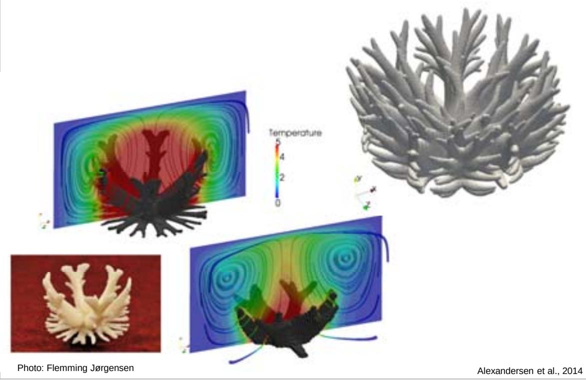
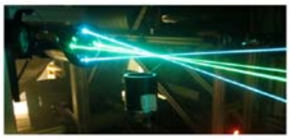
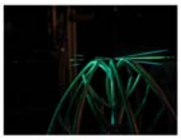


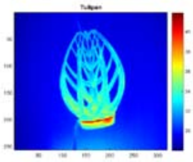
Photo: Flemming Jørgensen

Alexandersen et al., 2014

HyperCool project







DTU Mechanics, AT Lightning & ApioSoft Aps.

Niels Aage, Mechanical Engineering, Solid Mechanics   Technical University of Denmark