Role of helicity in three dimensional turbulence

Ganapati Sahoo and Luca Biferale

Department of Physics & INFN, University of Rome Tor Vergata, Italy.

Introduction

Turbulent flows are of interest in many scientific and industrial applications.

Such flows are governed by three dimensional Navier-Stokes equations. For incompressible flows the equations are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f},$$
$$\nabla\cdot\mathbf{u} = 0.$$

where u is the velocity, p is the pressure, ν is the kinematic viscosity, ρ is the density, f is the external forcing.

Direction of cascade of energy in a turbulent flow are determined by the invariants of the flow. In three dimensions the invariants are energy (E) and helicity (H).

$$E = \int \mathbf{u}(\mathbf{x})\cdot\mathbf{u}(\mathbf{x})d^3x \qquad H = \int \mathbf{u}(\mathbf{x})\cdot\omega(\mathbf{x})d^3x$$

Unlike energy helicity is a pseudo-scalar and changes sign under parity.

Since helicity is not positive-definite it does not allow the turbulent energy cascade from the small scales to the large scales.

However when helicity is made sign-definite by decimation of helical polarised waves of one sign an inverse cascade of energy from small scales to large scales is observed [1].

Fourier components of velocity can be decomposed into positive and negative helical modes

$$\mathbf{u}(\mathbf{k}) = \mathbf{u}^+(\mathbf{k}) + \mathbf{u}^-(\mathbf{k})$$

where $\mathbf{u}^+(\mathbf{k})$ and $\mathbf{u}^-(\mathbf{k})$ are the projections on the eigenvectors of the curl operator and could be obtained from the velocity field using a projector $P^{\pm}(\mathbf{k})$.

Decimated Navier-Stokes equations in Fourier space:

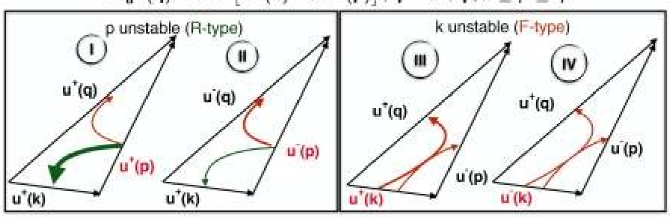
$$\partial_t \mathbf{u}^{\pm}(\mathbf{k},t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{N}_{u^{\pm}}(\mathbf{k},t) + \nu k^2\mathbf{u}^{\pm}(\mathbf{k},t) + \mathbf{f}^{\pm}(\mathbf{k},t)$$

The nonlinear term containing all triadic interactions

$$\mathbf{N}_{u^{\pm}}(\mathbf{k},t) = \mathcal{FT}(\mathbf{u}^{\pm}\cdot\nabla\mathbf{u}^{\pm} - \nabla p)$$

Triads in Navier-Stokes equations

$$\mathbf{N}_{u^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k})\cdot\nabla\mathbf{u}^{\pm}(\mathbf{p})\right];\ \mathbf{q} = \mathbf{k}+\mathbf{p};\ k \le p \le q$$

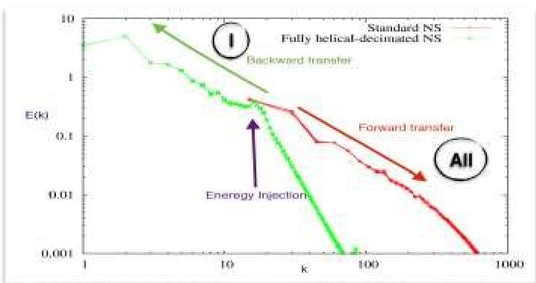


R-type: When large wavenumbers have same sign, middle one is unstable and could transfer energy to both small and large wavenumbers;

- predominantly to the smallest wavenumber if it has the same sign [Class-I (+, +, +)].
- mixed transfer if smallest wavenumber has the opposite sign [Class-II (+, -, -)].

F-type: When large wavenumbers have opposite sign, smallest one is unstable and could transfer energy only to large wavenumbers, for both Class-III (+, -, +) and Class-IV (-, -, +).

Direction of energy transfer



Triads with only u⁺, i.e. Class-I, lead to reversal of energy cascade.

Stability of inverse cascade

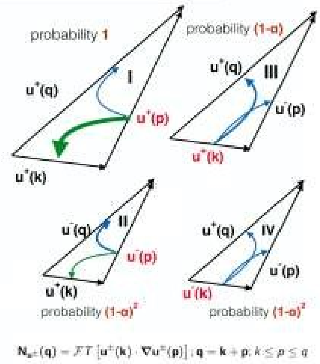
We removed a fraction (α) of the negative helical modes from the dynamics using a projection at every time step in our simulations to study the efficiency of the inverse energy transfer in presence of triads other than Class-I.

What happens in between??
when we give different weights to different class of triads…

- Modified projection operator:

  $\mathcal{P}^{+}_{\alpha}(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^{+}(\mathbf{k},t) + \theta_{\alpha}(\mathbf{k})\mathbf{u}^{-}(\mathbf{k},t)$

  where $\theta_{\alpha}(\mathbf{k})$ is 0 with probability α and is 1 with probability $1-\alpha$.
- We consider triads of Class-I with probability 1, Class-III with probability $1-\alpha$ and Class-II and Class-IV with probability $(1-\alpha)^2$.
- $\alpha = 0 \rightarrow$ Standard Navier-Stokes.
  $\alpha = 1 \rightarrow$ Fully helical-decimated NS.
- Critical value of α at which forward cascade of energy stops? alternatively, inverse cascade of energy stops if forced at small scales.



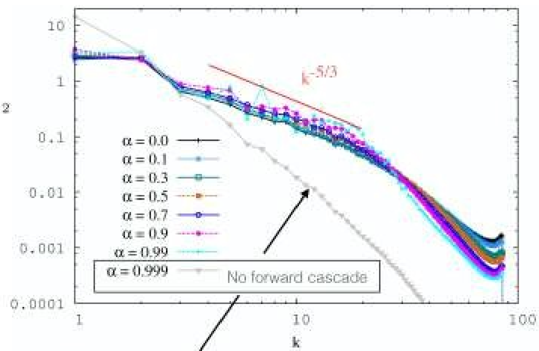
$\mathbf{N}_{u^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k})\cdot\nabla\mathbf{u}^{\pm}(\mathbf{p})\right];\ \mathbf{q} = \mathbf{k}+\mathbf{p};\ k \le p \le q$

Results

Energy spectra

$$E(k) = \sum_{|\mathbf{k}|\in[k,k+1]} |\mathbf{u}(\mathbf{k})|^2$$

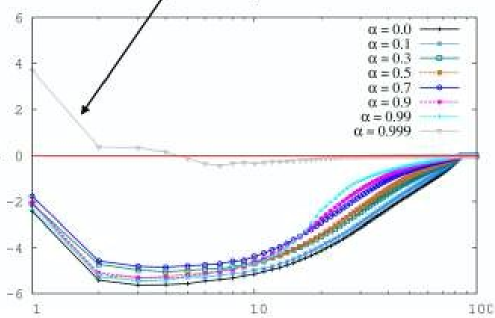
shows robustness of forward energy cascade.



Flux of energy

$$\Pi_E(k) = \sum_{|k'|<k} \hat{u}^*_{k'}\cdot\hat{\mathbf{N}}_{k'}$$

is unaffected by decimation.



Conclusions

- As we increase decimation of the modes with negative helicity (α) the contribution of triads leading to inverse energy transfer grows.
- The inverse energy cascade is very fragile; the critical value of α is very close to 1.
- Forward energy cascade is very robust in three dimensional turbulence; presence of few helical modes of different sign helps to transfer energy forward.
- The flux of energy for forward transfer of energy is independent of degree of decimation (α).
- A strong tendency to recover parity invariance is observed due to super-efficient negative helical modes to maintain a constant energy flux in presence of explicit breaking of mirror symmetry (α>0).

References

[1] Inverse energy cascade in three-dimensional isotropic turbulence, L Biferale, S Musacchio, and F Toschi. Phys. Rev. Lett. 108, 164501 (2012)

[2] Role of helicity for large-and small-scales turbulent fluctuations, G Sahoo, F Bonaccorso, and L Biferale. Phys. Rev. E 92, 051002 (R) (2015).

[3] Disentangling the triadic interactions in Navier-Stokes equations, G Sahoo and L Biferale. Eur. Phys. J. E 38, 114 (2015).

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