

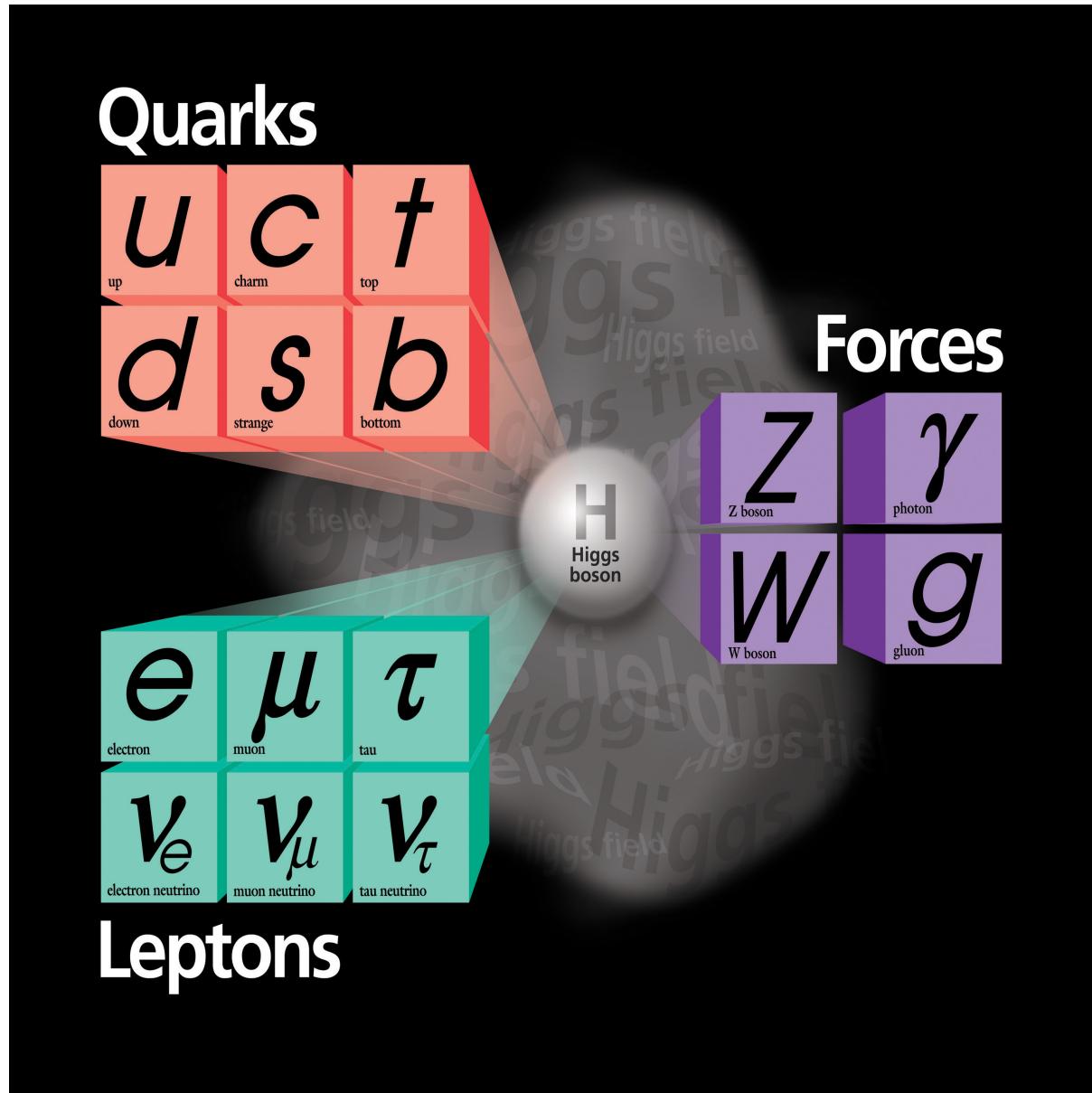
Singlet physics the missing link to precision lattice QCD

Karl Jansen



- Introduction: quarks and gluons
- Gluons at work:
 - The (mysterious) mass of the η' -meson
 - How much does the proton know about the strange quark?
- Conclusion

The standard model of elementary particle interaction



*with the (most likely)
discovery of the
Higgs boson
the standard model
is complete*

Quark-Gluon sector of the standard model

Quarks are the fundamental constituents of nuclear matter

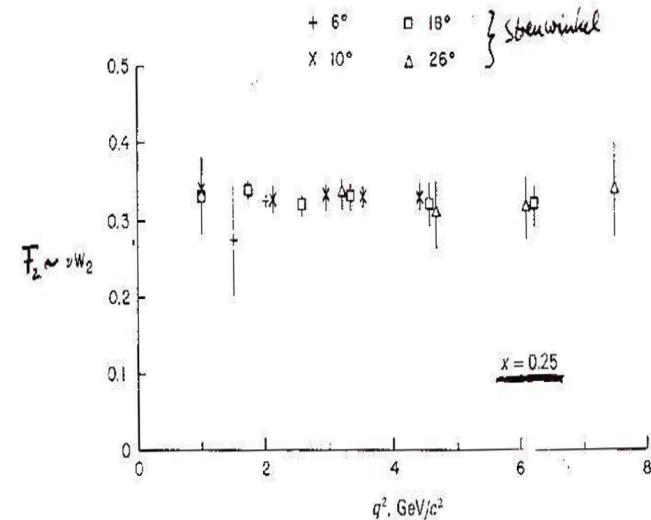
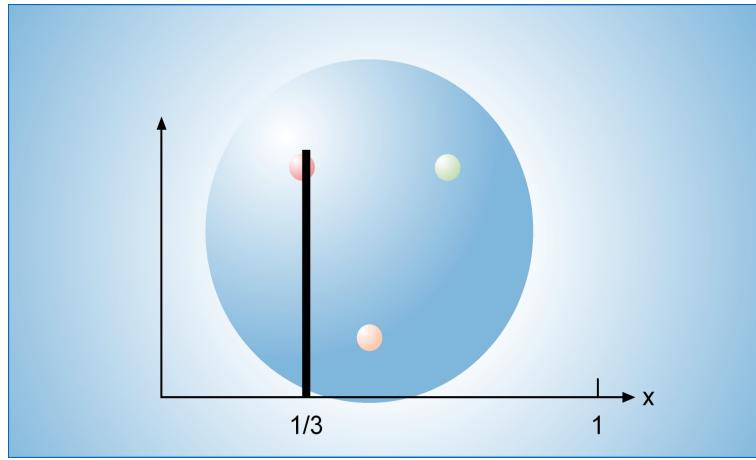


Fig. 7.17 vW_2 (or F_2) as a function of q^2 at $x = 0.25$. For this choice of x , there is practically no q^2 -dependence, that is, exact "scaling". (After Friedman and Kendall 1972.)

Friedman and Kendall, 1972)

$$f(x, Q^2) \Big|_{x \approx 0.25, Q^2 > 10 \text{ GeV}} \text{ independent of } Q^2$$

(x momentum of quarks, Q^2 momentum transfer)

Interpretation (Feynman): scattering on single quarks in a hadron
→ (Bjorken) scaling

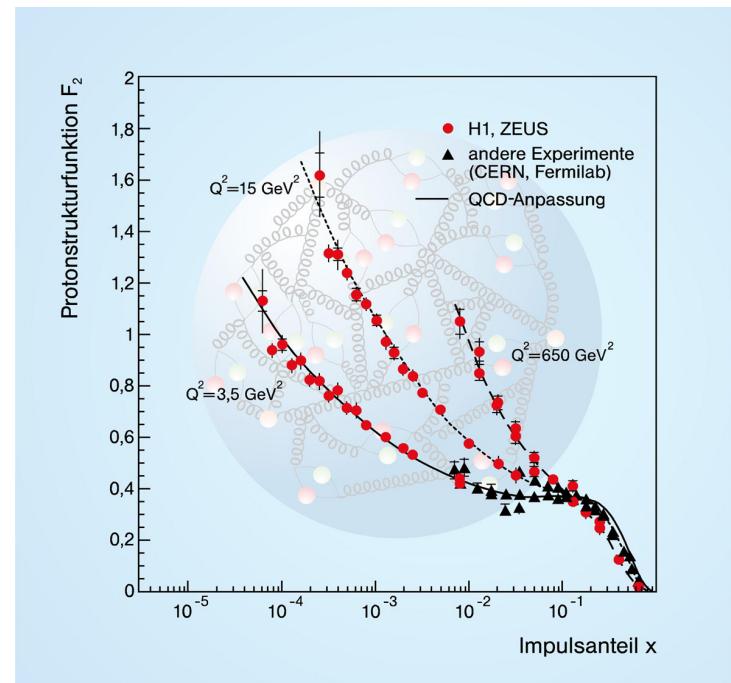
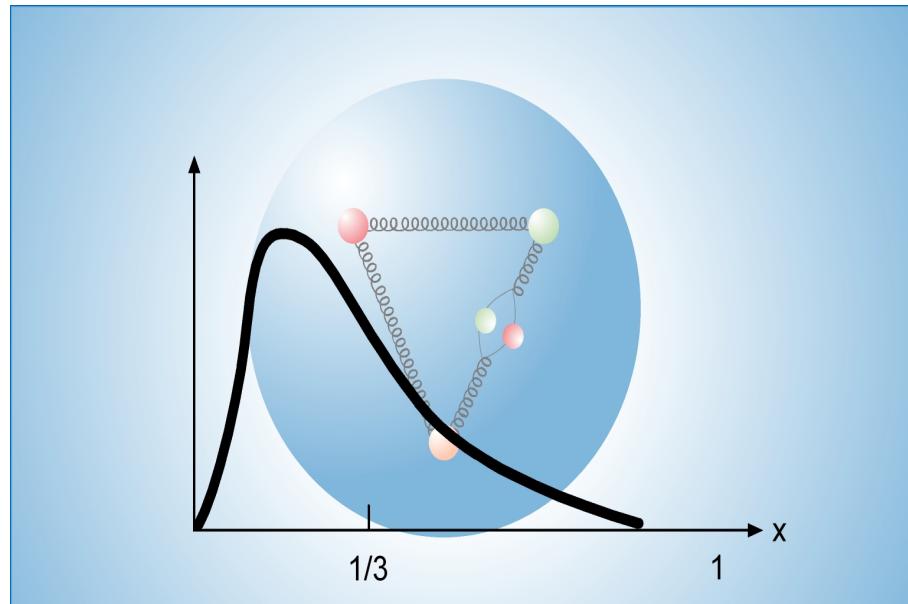
Quantum Fluctuations and the Quark Picture

analysis in perturbation theory

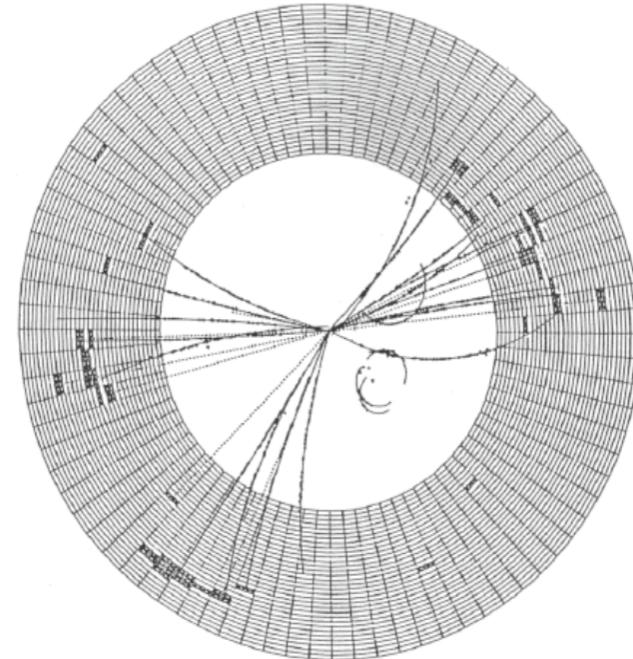
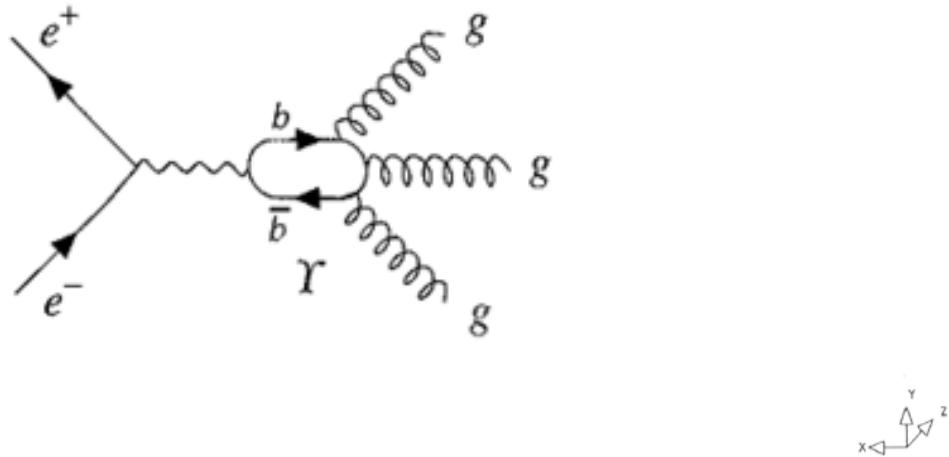
$$\int_0^1 dx f(x, Q^2) = 3 \left[1 - \frac{\alpha_s(Q^2)}{\pi} - a(n_f) \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 - b(n_f) \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 \right]$$

– $a(n_f), b(n_f)$ calculable coefficients

deviations from scaling → determination of strong coupling



The discovery of the gluon



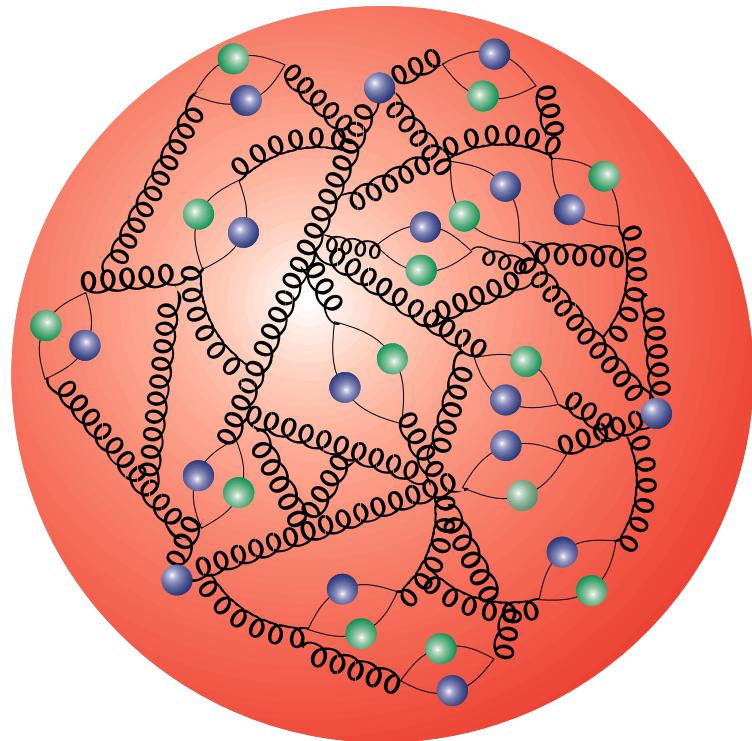
*** SUMS (GeV) *** PTOT 35.768 PTRANS 29.964 PLONG 15.788 CHARGE -2
TOTAL CLUSTER ENERGY 15.169 PHOTON ENERGY 4.893 NR OF PHOTONS 11

Feynman diagram for 3 gluon decay
each gluon generates a jet

3-jet event at Jade-detector
DESY (1978)

Why Perturbation Theory fails for the Strong Interaction

- situation becomes incredibly complicated
 - value of the coupling (expansion parameter)
 $\alpha_{\text{strong}}(1\text{fm}) \approx 1$
- ⇒ need different (“exact”) method
- ⇒ has to be non-perturbative
- Wilson’s Proposal (1974): Lattice Quantum Chromodynamics



Schwinger model: 2-dimensional Quantum Electrodynamics

(Schwinger 1962)

Quantization via Feynman path integral

$$\mathcal{Z} = \int \mathcal{D}A_\mu \mathcal{D}\Psi \bar{\mathcal{D}}\bar{\Psi} e^{-S_{\text{gauge}} - S_{\text{ferm}}}$$

Fermion action

$$S_{\text{ferm}} = \int d^2x \bar{\Psi}(x) [D_\mu + m] \Psi(x)$$

gauge covariant derivative

$$D_\mu \Psi(x) \equiv (\partial_\mu - ig_0 A_\mu(x)) \Psi(x)$$

with A_μ gauge potential, g_0 bare coupling

$$S_{\text{gauge}} = \int d^2x F_{\mu\nu} F_{\mu\nu}, \quad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

equations of motion: obtain classical Maxwell equations

Lattice Schwinger model

introduce a **2-dimensional** lattice with
lattice spacing a

fields $\Psi(x)$, $\bar{\Psi}(x)$ on the lattice sites

$x = (t, \mathbf{x})$ integers

discretized fermion action

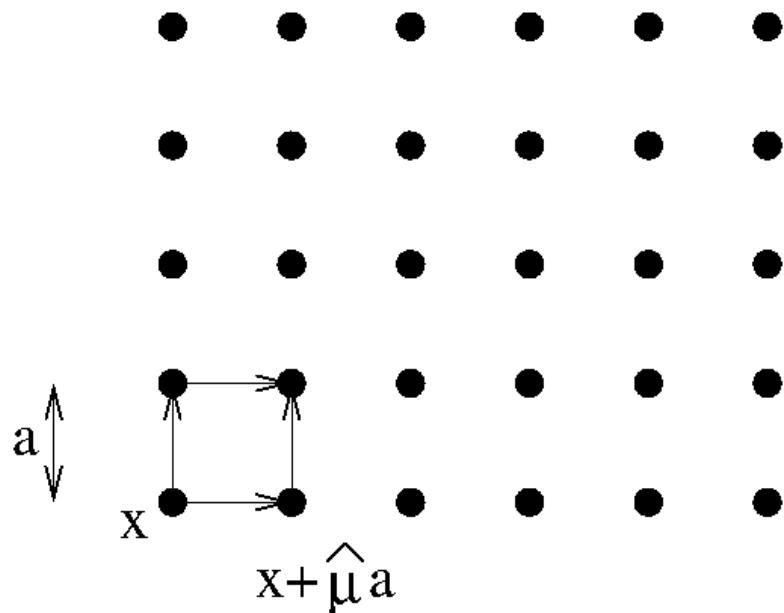
$$S \rightarrow a^2 \sum_x \bar{\Psi} [\gamma_\mu \partial_\mu - r \underbrace{\nabla_\mu^* \nabla_\mu}_{\nabla_\mu^* \nabla_\mu} + m] \Psi(x)$$

$$\partial_\mu = \frac{1}{2} [\nabla_\mu^* + \nabla_\mu]$$

discrete derivatives

$$\nabla_\mu \Psi(x) = \frac{1}{a} [\Psi(x + a\hat{\mu}) - \Psi(x)] , \quad \nabla_\mu^* \Psi(x) = \frac{1}{a} [\Psi(x) - \Psi(x - a\hat{\mu})]$$

second order derivative \rightarrow remove doubler \leftarrow break chiral symmetry

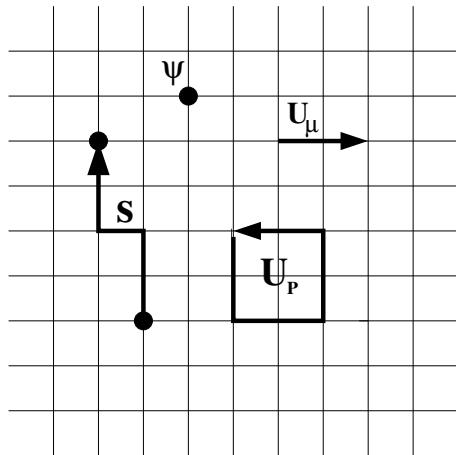


Implementing gauge invariance

Wilson's fundamental observation: introduce Paralleltransporter connecting the points x and $y = x + a\hat{\mu}$:

$$U(x, \mu) = e^{iaA_\mu(x)} \in U(1)$$

\Rightarrow lattice derivative: $\nabla_\mu \Psi(x) = \frac{1}{a} [U(x, \mu)\Psi(x + \mu) - \Psi(x)]$



$$U_p = U(x, \mu)U(x + \mu, \nu)U^\dagger(x + \nu, \mu)U^\dagger(x, \nu)$$

$$\rightarrow F_{\mu\nu}F^{\mu\nu}(x) \quad \text{for} \quad a \rightarrow 0$$

$$S = a^2 \sum_x \left\{ \beta [1 - \text{Re}(U_{(x,p)})] + \bar{\psi} [\text{m}_0 + \frac{1}{2} \{ \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - a \nabla_\mu^* \nabla_\mu \}] \psi \right\}$$

Partition functions (pathintegral) with Boltzmann weight (action) S

$$\mathcal{Z} = \int_{\text{fields}} e^{-S}$$

Physical Observables

expectation value of physical observables \mathcal{O}

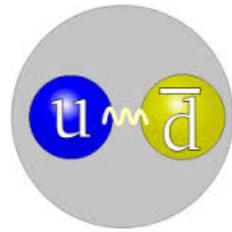
$$\underbrace{\langle \mathcal{O} \rangle = \frac{1}{Z} \int_{\text{fields}} \mathcal{O} e^{-S}}$$

↓ lattice discretization

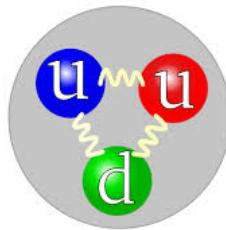
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Composition of mesons and nucleons

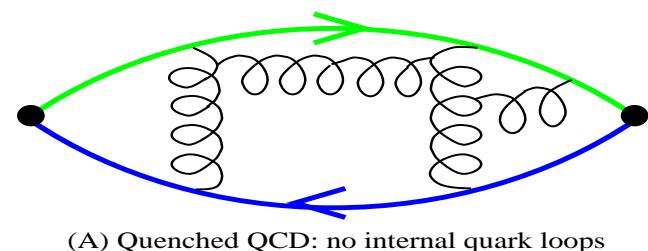


Pion

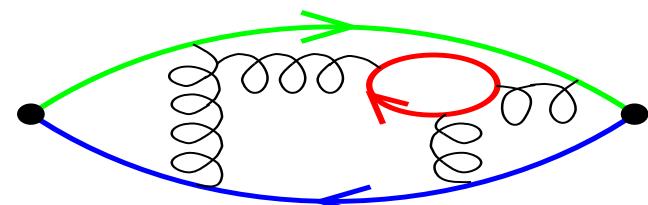


Proton

representation



(A) Quenched QCD: no internal quark loops



(B) full QCD

An unexpectedly large mass

meson	composition	approx. mass
K^0	$d + s$	498MeV
K^+	$u + s$	494MeV
K^-	$u + s$	494MeV
η	$u + u + s$	548MeV
η'	$u + d + s$	958MeV

naively: derive masses of η and η' from quark-content of K 's

$m_{u,d} \approx 5\text{MeV}$, $m_s \approx 100\text{MeV}$

side remark: $m_{\text{meson}} \gg m_{\text{quark}} \rightarrow$ binding energy!

expectation

$$m_\eta \approx m_{\eta'} \approx m_K + 2 \cdot m_u \approx 500\text{MeV}$$

find $m_{\eta'} = 958\text{MeV}$

\rightarrow clear contradiction

Topology

Veneziano-Witten relation

relation of flavour singlet η' mass to topological susceptibility

$$\frac{f_\pi^2}{2N_f} (m_\eta^2 + m_{\eta'}^2 - 2m_K^2) = \chi_{\text{top}}^{\text{YM}}$$

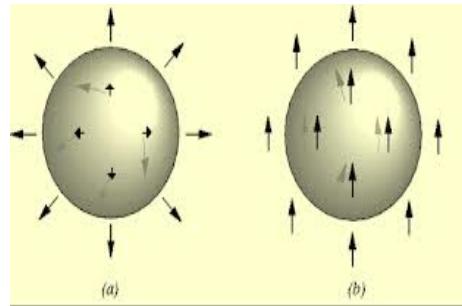
$$\chi_{\text{top}}^{\text{YM}} = Q_{\text{topo}}^2$$

Q_{topo} **topological charge** *in pure gluonic theory*

→ think of topological defect

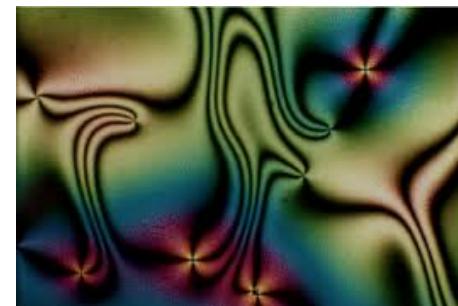
Topological charge

topological defect in liquid crystals (screens)

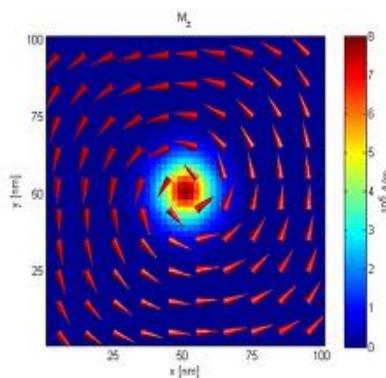


defect (schematically)

spin vortex



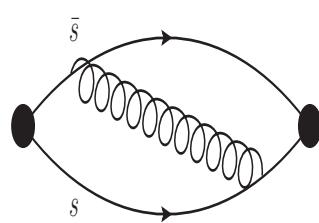
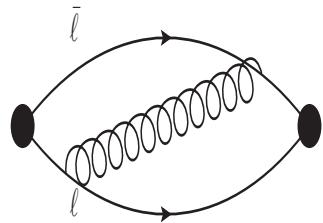
defect (observed)



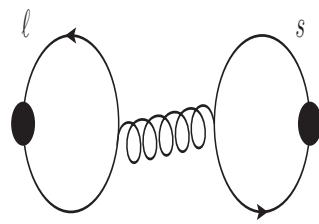
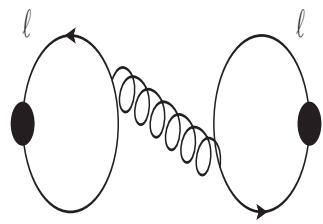
- vortex in spin models
- topological charge in QCD
- related to *homotopy group of the $n -$ sphere* (principle bundle)

Singlet, dis-connected contributions

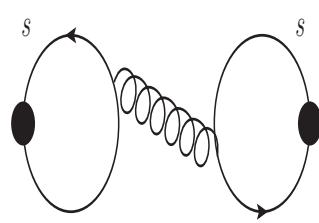
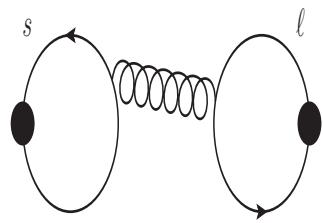
η' is singlet state



- meson with quark interaction



- pure gluon interaction



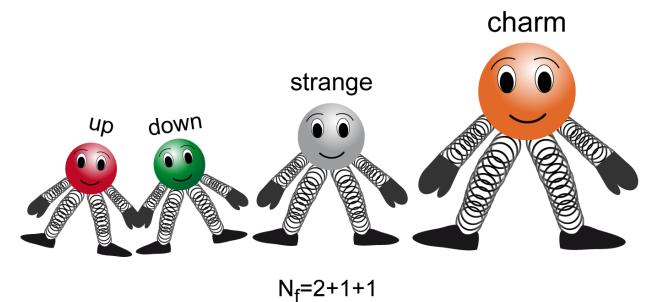
- test of gluons at work

t

t'

t

t'



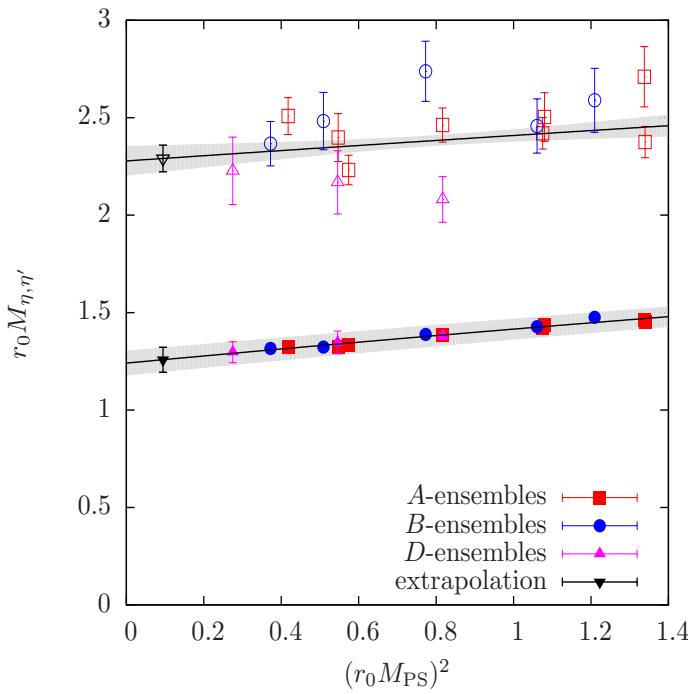
Practical usage: application for $N_f = 2 + 1 + 1$ maximally twisted mass fermions

Ensemble	β	lattice	a [fm]	$a\mu_l$	μ_l [MeV]	κ_c	L [fm]
A30.32	1.90	$32^3 \times 64$	0.086	0.003	13	0.163272	2.75
A50.32	1.90	$32^3 \times 64$	0.086	0.005	22	0.163267	2.75
A60.24	1.90	$24^3 \times 48$	0.086	0.006	26	0.163265	2
B25.32	1.95	$32^3 \times 64$	0.078	0.0025	13	0.161240	2.5
B35.32	1.95	$32^3 \times 64$	0.078	0.0035	18	0.161240	2.5
B55.32	1.95	$32^3 \times 64$	0.078	0.0055	28	0.1612360	2.5
B75.32	1.95	$32^3 \times 64$	0.078	0.0075	38	0.1612320	2.5
B85.24	1.95	$24^3 \times 48$	0.078	0.0085	45	0.1612312	2.0
D15.48	2.1	$48^3 \times 96$	0.0612	0.0015	9	0.156361	3.0
D20.48	2.1	$48^3 \times 96$	0.0612	0.0020	12	0.156357	3.0
D30.48	2.1	$48^3 \times 96$	0.0612	0.0030	19	0.156355	3.0
D45.32	2.1	$48^3 \times 96$	0.0612	0.0045	28	0.156315	2.0

ensembles are freely available on [ILDG](#)

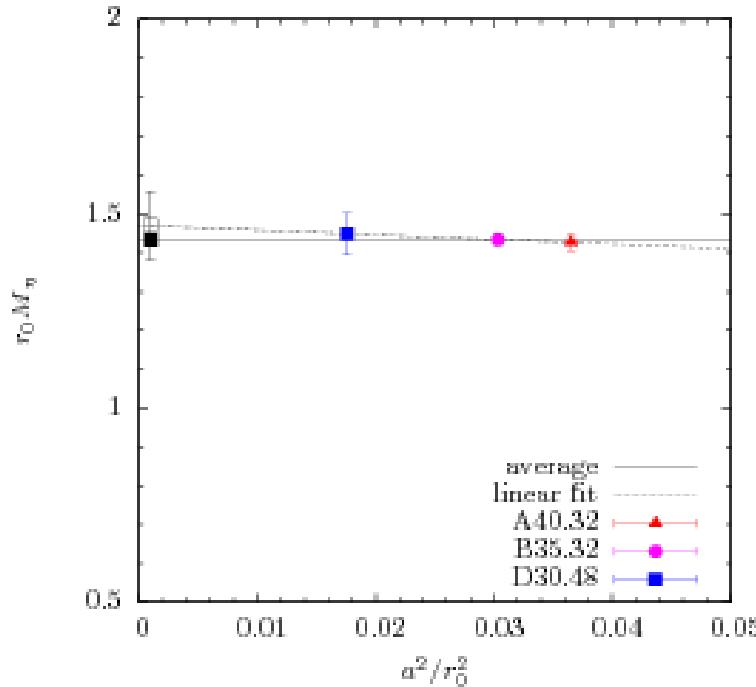
The left-hand side: meson masses

(Ottnad, Michael, Reker, Urbach)



chiral extrapolation

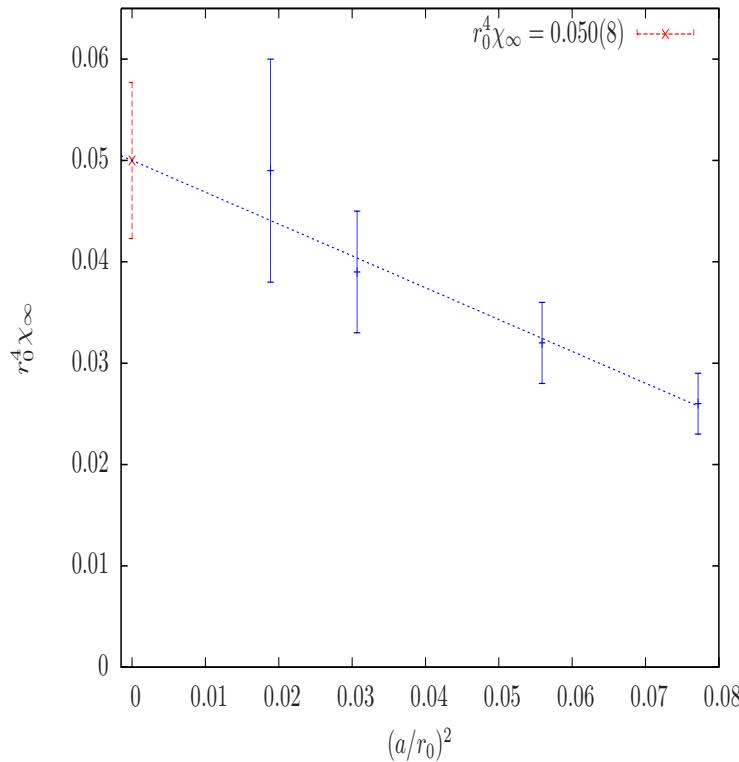
- η' mass noisy but non-zero
- can obtain η and η' masses



continuum extrapolation

The right-hand side: topological susceptibility

(Garcia Ramos, Cichy, K.J.)



- dedicated quenched effort
- learn about continuum limit of χ_{top}

- consistent with $O(a^2)$ continuum limit scaling
- have both sides of Veneziano-Witten relation *in the continuum limit*

Veneziano-Witten relation

- relation of flavour singlet mass to *quenched* topological susceptibility

$$\frac{f_\pi^2}{2N_f} (m_\eta^2 + m_{\eta'}^2 - 2m_K^2) = \chi_{\text{top}}^{\text{YM}}$$

- formula needs limit: $N_f/N_c \rightarrow 0$
- Hence, test in full QCD ($N_f = 4, N_c = 3$) in non-trivial
- obtain (preliminary) ($M_K = 497.648(22)\text{MeV}, f_\pi = 130.41(4)\text{MeV}$) input

am_η [MeV]	$am_{\eta'}$ [MeV]	$\chi_{\text{top}}^{\text{YM}}$
551(8)	1001(55)	$(187(7)\text{MeV})^4$

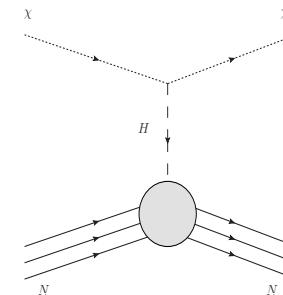
$$\frac{(af_\pi)^2}{4N_f} ((am_\eta)^2 + (am_{\eta'})^2 - 2(am_K)^2) = (184(6)\text{MeV})^4$$

→ find very nice agreement

The strange quark content of the nucleon

(Alexandrou, Constantinou, Drach, Dinter, Frezzotti, Herdoiza, Koutsou, Rossi, Vaquero, K.J.)

- neutralino in supersymmetric models candidate for dark matter
- interaction with nucleon most strongly through the strange quark content via the Higgs boson exchange diagram



spin independend cross section:

$$\sigma_{\text{SI}} \propto \sum_q \frac{\langle N | \bar{q} q | N \rangle}{m_N} ; q = u, d, s, c$$

⇒ cross section proportional to quark content

Study here: strange quark content $\langle N | \bar{s} s | N \rangle$

The problem

spin independent cross section also strongly dependend on pion-nucleon sigma term $\sigma_{\pi N} \equiv m_l \langle N | \bar{u}u + \bar{d}d | N \rangle$

Varying $48 \text{ MeV} < \Sigma_{\pi N} < 80 \text{ MeV}$

\Rightarrow cross section changes by an order of magnitude

$\Sigma_{\pi N}$ connected to y_N parameter $y_N = \frac{2 \langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u | N \rangle + \langle N | \bar{d}d | N \rangle}$

relation: $y_N = 1 - \sigma_0 / \Sigma_{\pi N}$

$$\sigma_0 = m_q \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle, \quad m_q = (m_u + m_d)/2$$

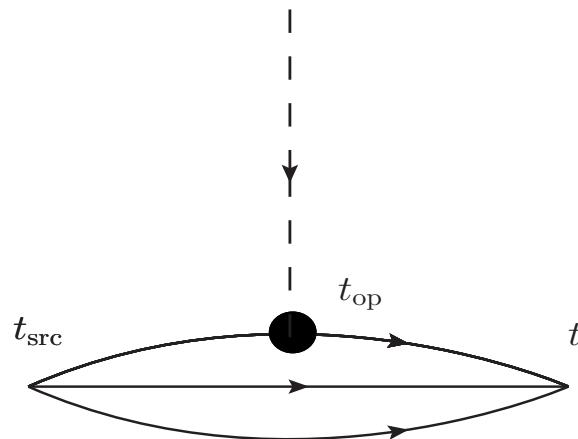
from effective field theory (χ PT):

$$y_N^{\text{GLS}} = 0.20(21), \quad y_N^{\text{GWU}} = 0.44(13), \quad y_N^{\text{AMO}} = 0.39(14)$$

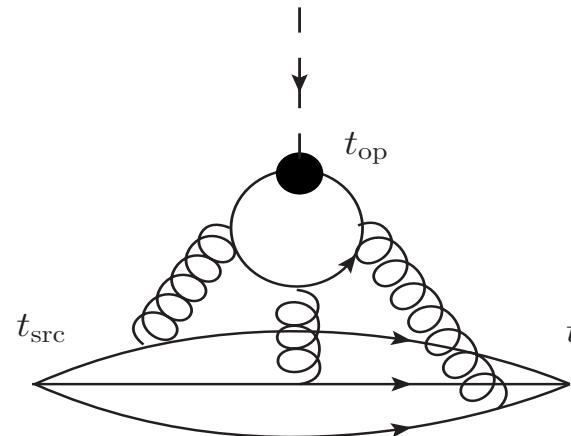
(GLS: Gasser et.al. (1991), GWU: Pavan et.al. (2002), AMO: Alarcon et.al. (2012))

\rightarrow either compatible with zero or quite large \rightarrow large cross-section

The dis-connected (singlet) contribution



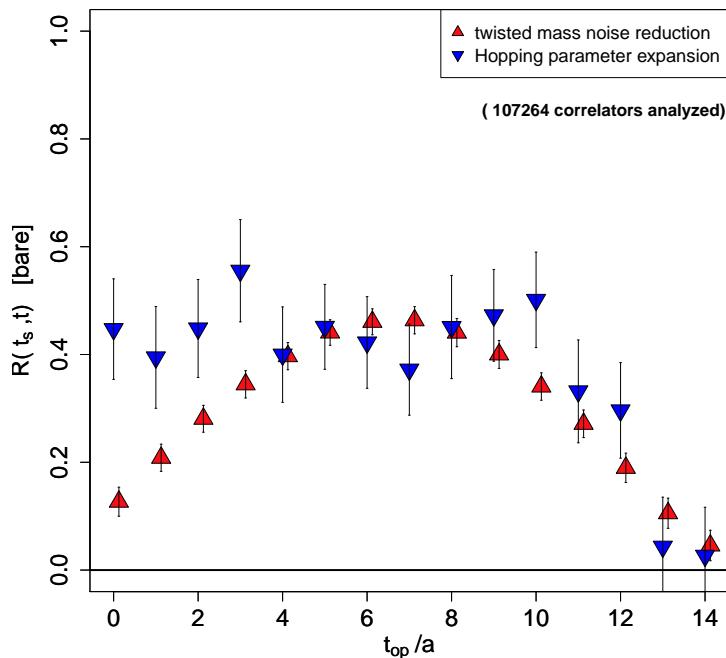
connected



dis-connected

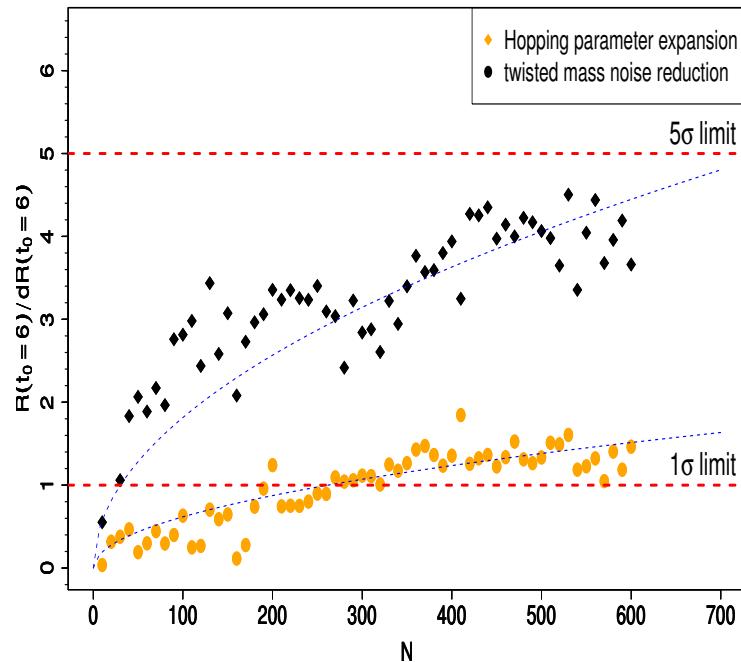
- connected contribution can be computed precisely
- dis-connected only interact via gluons
- strange quark content only dis-connected contributions
 - need a huge (unrealistic) statistics
 - ⇒ lattice calculation of strange quark content very difficult
- another example of gluons at work
- our setup: avoids mixing in renormalization

Twisted mass fermions: special noise reduction technique



- employed a huge $O(100\ 000)$ measurements
(typical: 1000 measurements)
- plateau of $R(t_s, t) \propto y_N$
- hopping parameter expansion already better than straightforward calculation
- substantially reduced error

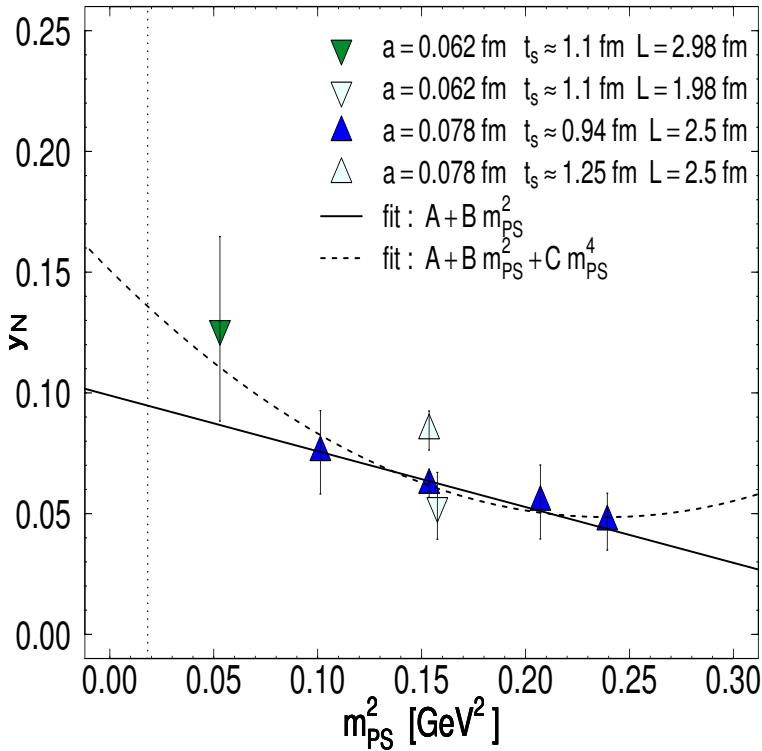
Twisted mass fermions: special noise reduction technique



- signal/error for strange quark content
- N number of configurations used

- typical statistics normally $N \approx 100 - 200$
- standard calculation very (too?) difficult
- using techniques for twisted mass fermions
 - can obtain a signal with reasonable statistics

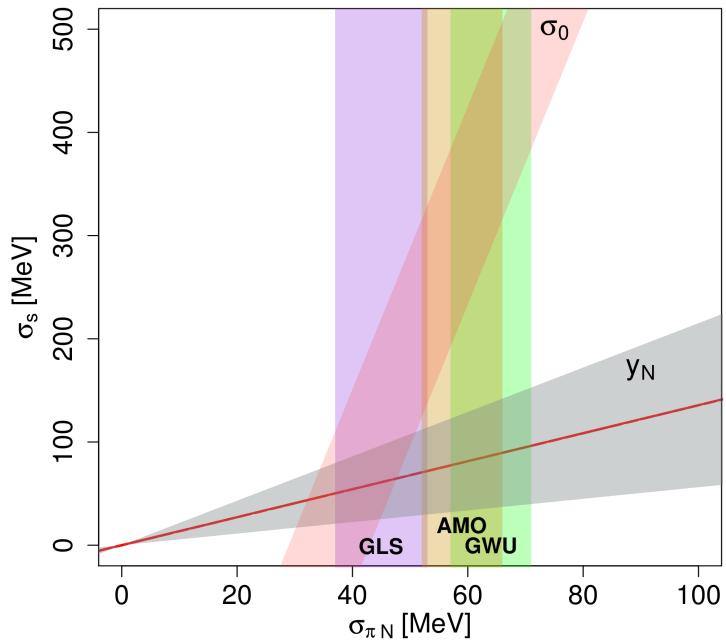
Chiral extrapolation of y_N



- result: $y_N = 0.099(12)(52)(22)(9)$
- errors: (statistical)(chiral)(excited)(discretization)
- obtain: $y_N = 0.099(58)$
- lattice result with all systematic errors included

- chiral extrapolation error dominates → need physical point
- value significantly smaller than from effective field theory
- Higgs-boson Wimp cross-section might be unexpectedly small

Impact of y_N on σ -terms



- $y_N = 1 - \sigma_0 / \Sigma_{\pi N}$

- our result constraints strange quark content $\sigma_s = \langle N | \bar{s}s | N \rangle$

Summary

- Showed two examples of gluons at work
 - the mass of the η' meson
 - the strange quark content of the nucleon
- results became possible through
 - special twisted mass noise reduction techniques
 - improved renormalization pattern
- allowed to test the fundamental Veneziano-Witten relation
- calculation of dis-connected diagrams for other observables in sight
- Prace project has been extremely helpful