



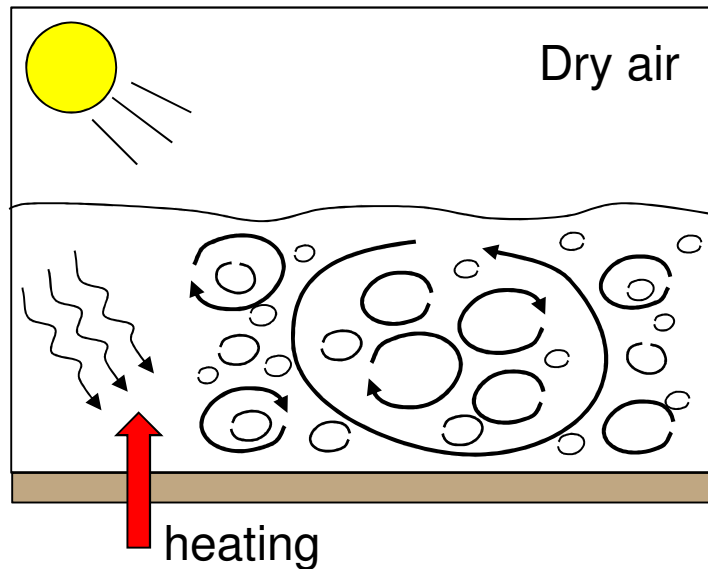
Direct simulation of turbulent entrainment across density interfaces

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Harm Jonker², Gary Hunt¹, Peter Sullivan³, Ned Patton³

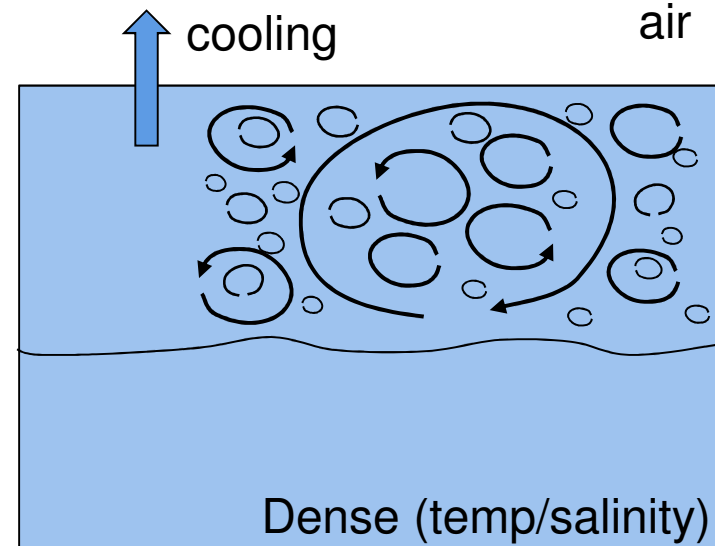
¹Imperial College London, UK ²Delft University of Technology, NL ³NCAR USA

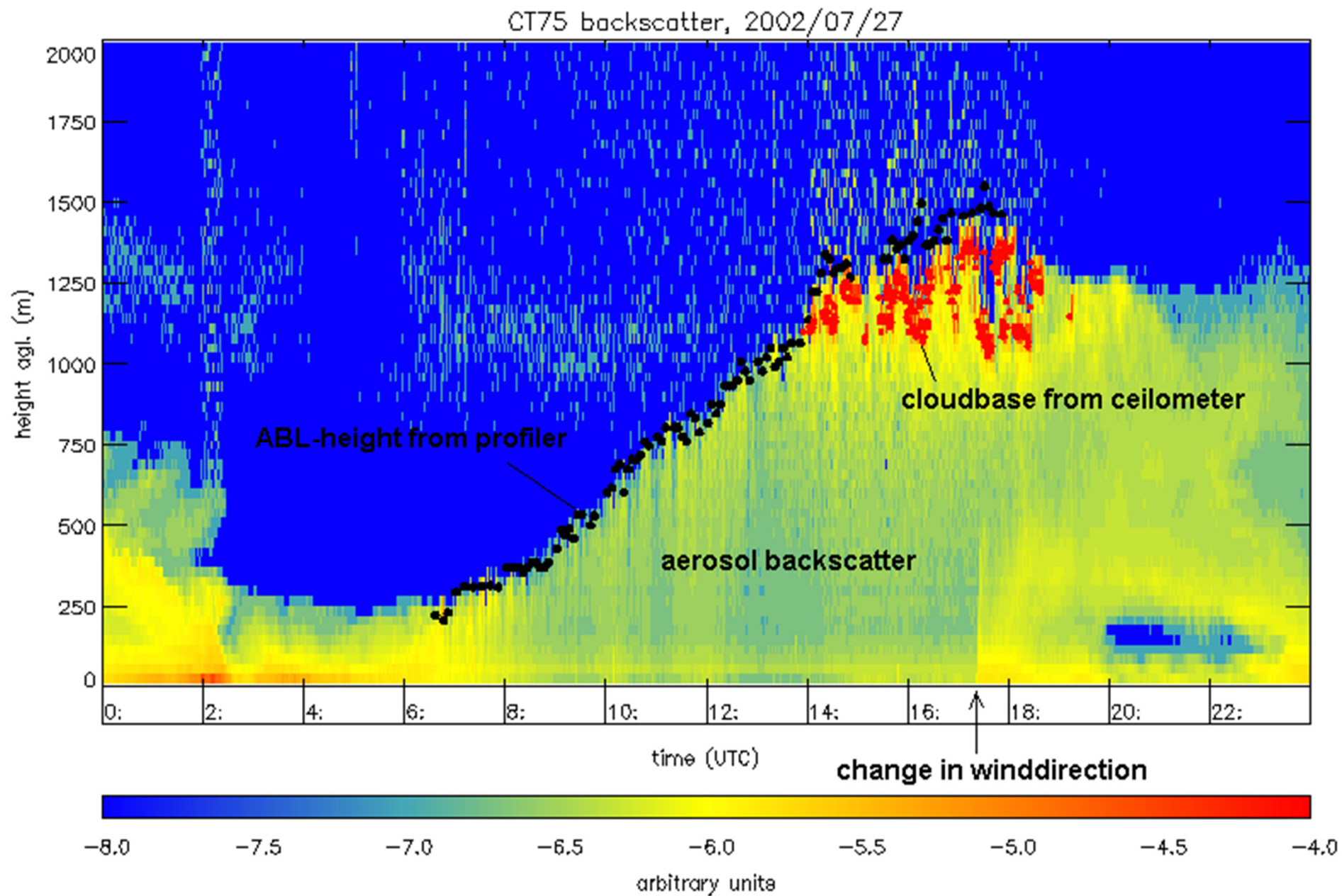
Entrainment in geophysical flows



Atmospheric boundary layer
Daytime convection

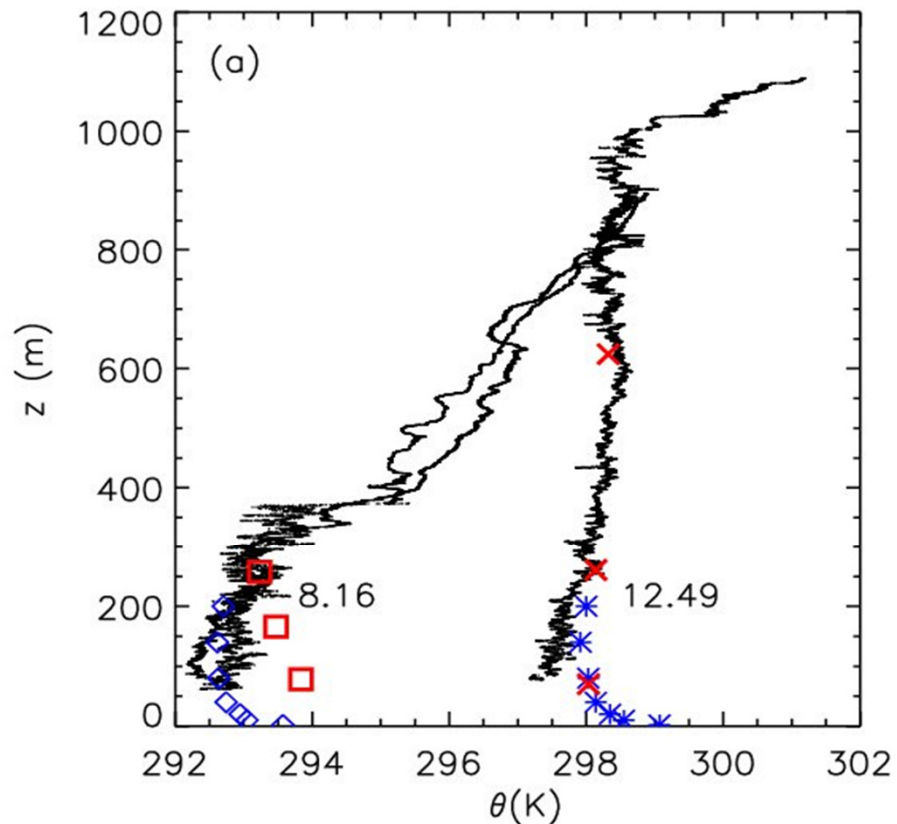
Marine boundary layer
Night-time convection



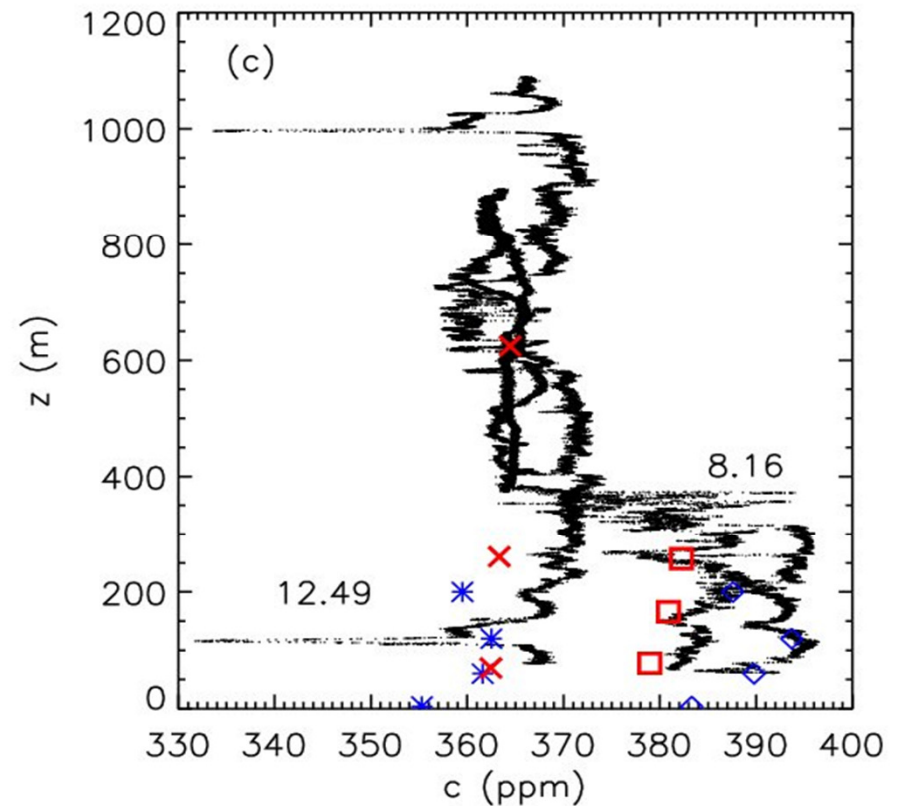


Measurements in the ABL

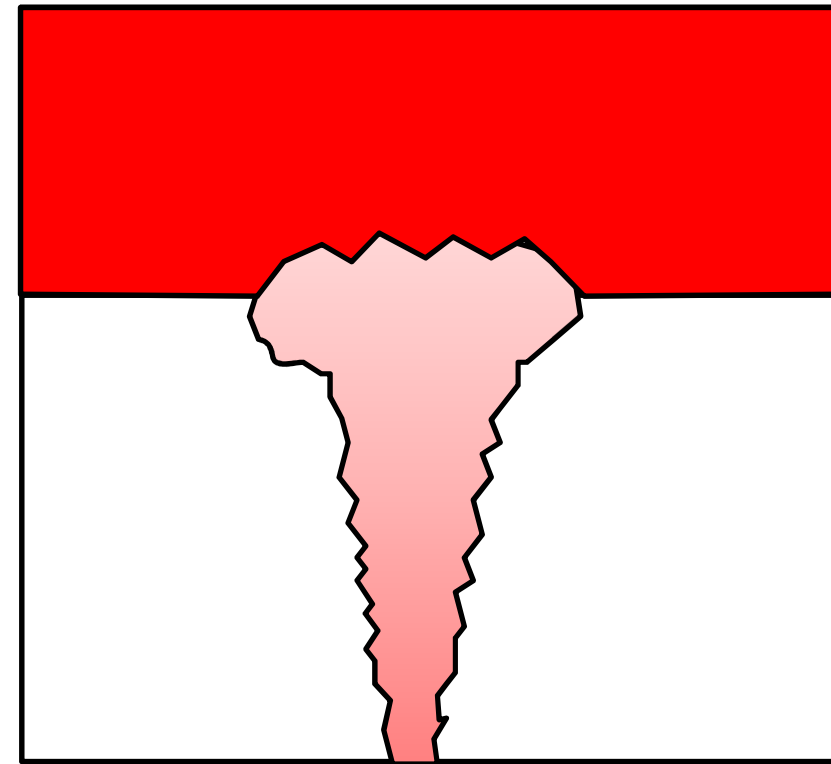
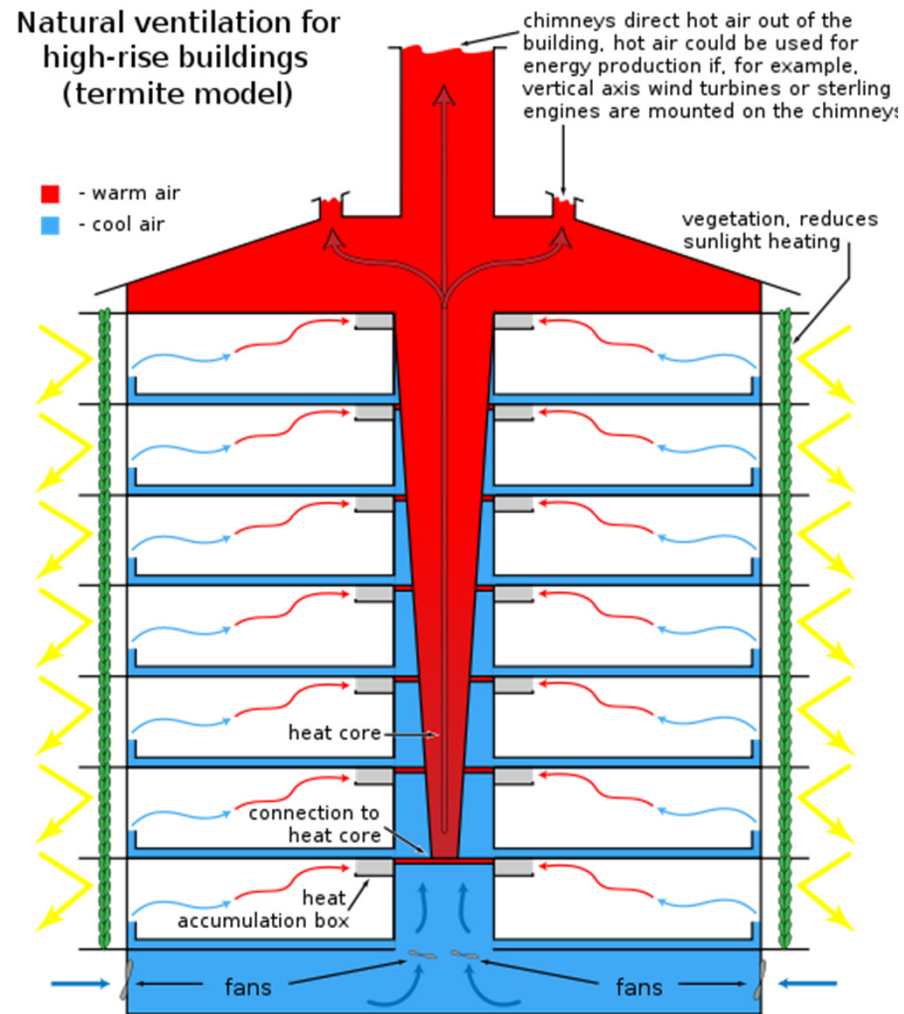
Potential temperature



Carbon dioxide

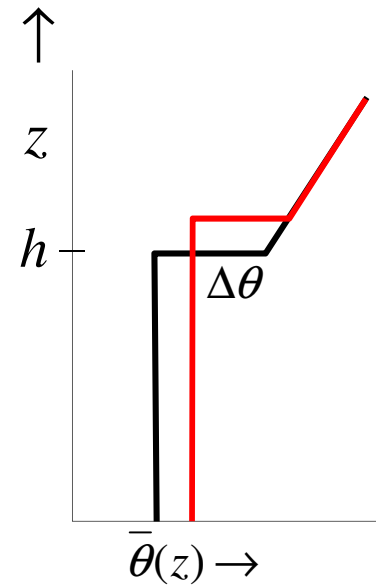
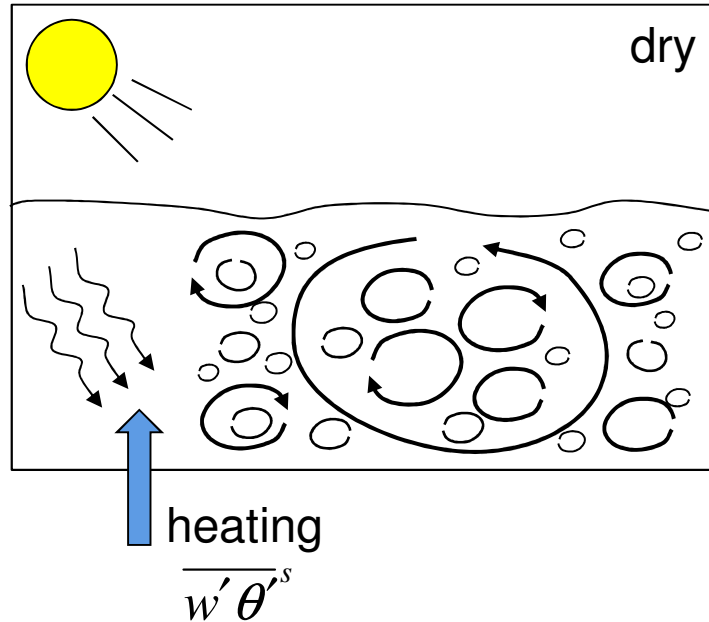


Entrainment in engineering flows



Occupant, Equipment, ...

Governing parameters – homogeneous case



Velocity scale

$$w_* = \left(\frac{g}{\theta_0} \overline{w'\theta'^s} h \right)^{1/3}$$

Reynolds number: $\text{Re} = \frac{w_* h}{\nu}$

Péclet number: $\text{Pe} = \frac{w_* h}{\kappa}$

Richardson number: $\text{Ri} = \frac{g}{\theta_0} \frac{\Delta\theta h}{w_*^2}$

Prandtl number: $\text{Pr} = \frac{\nu}{\kappa}$

Entrainment laws

- The entrainment velocity w_e is defined as the time rate of change of the mixed layer depth:

$$w_e = \frac{dh}{dt}$$

- We expect there to be a unique entrainment law E :

$$\frac{w_e}{w_*} = E(\text{Ri}, \text{Re}, \text{Pe})$$

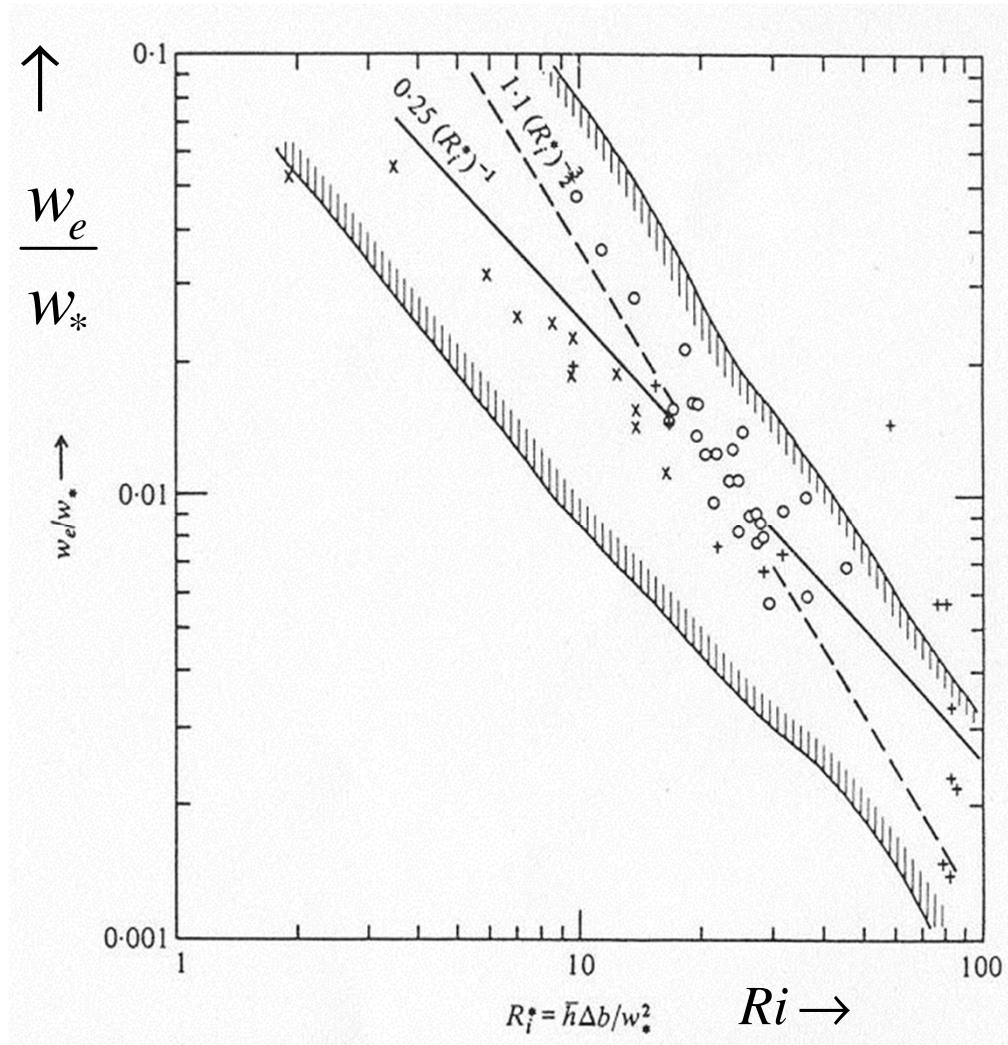
- which in the case of $\text{Re}, \text{Pe} \rightarrow \infty$ (asymptotic state) becomes

$$\frac{w_e}{w_*} = E(\text{Ri})$$

How to obtain the entrainment law?

- Atmospheric Observations/field experiments
 - The real thing!
 - But ... incomplete information, as is (no control), reproducibility, parameter studies impossible
- Numerical Simulation: LES (RANS, ...)
 - Complete information, excellent control (BCs, forcing), reproducible, parameter studies
 - But ... not real, relies on turbulence closures.
- Laboratory Experiments (convection tank)
 - Reasonable amount of information, reasonable control, reasonable reproducibility, parameter studies possible, it is real
 - Viscous effects play a role

Laboratory experiments with a heated water tank



$$Re=10^3$$

$$Pr = 7$$

$$w_e = \frac{dh}{dt}$$

$$\frac{w_e}{w_*} = ARi^{-1}$$

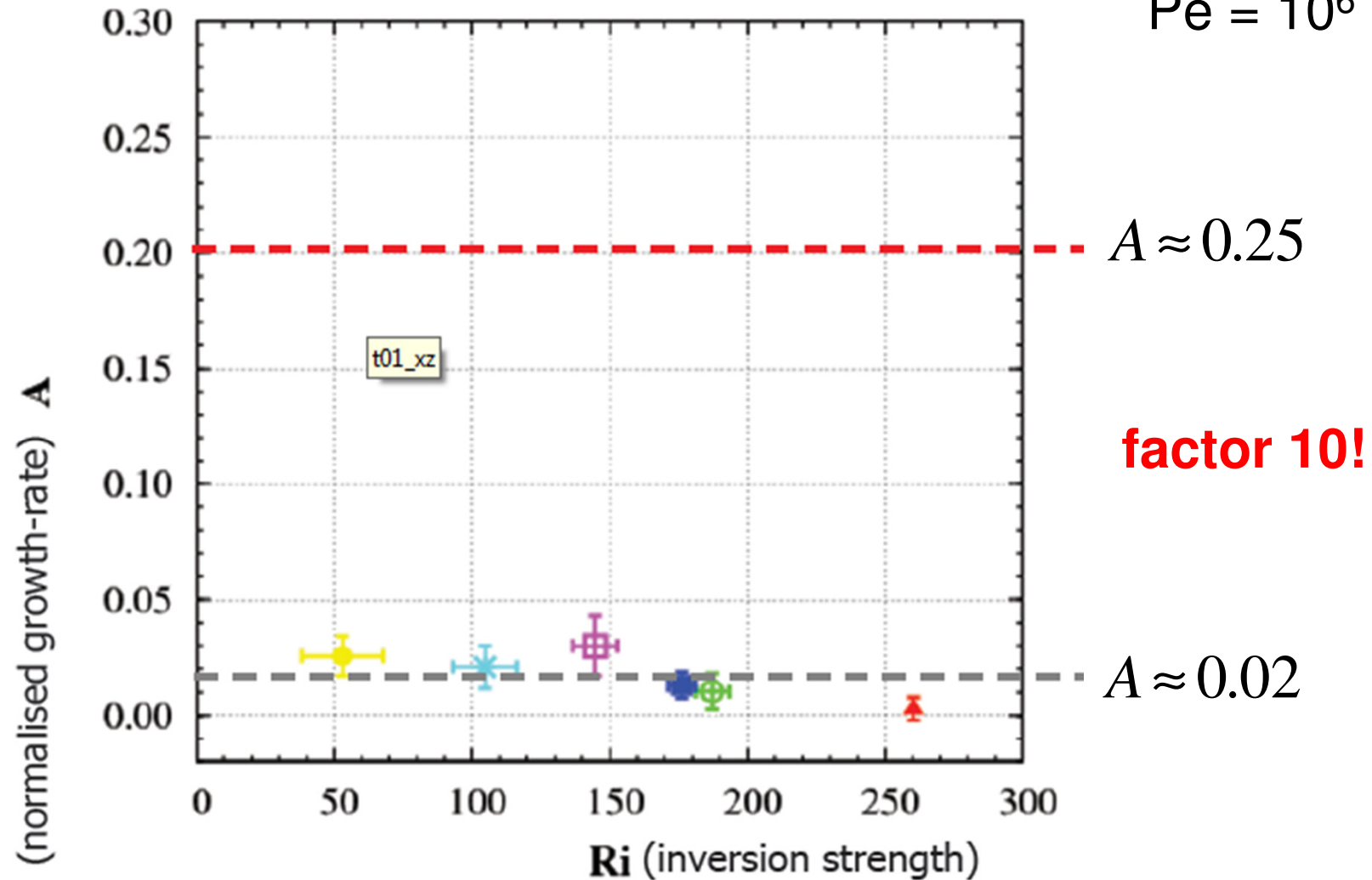
$$A=0.25$$

(Deardorff, Willis and Stockton, JFM 1980)

Results of Delft saline tank

$$Re = 10^3$$

$$Pe = 10^6$$



(Jonker *et al.*, in preparation)

The case for direct numerical simulation

- **Ab initio simulation of fluid behaviour**

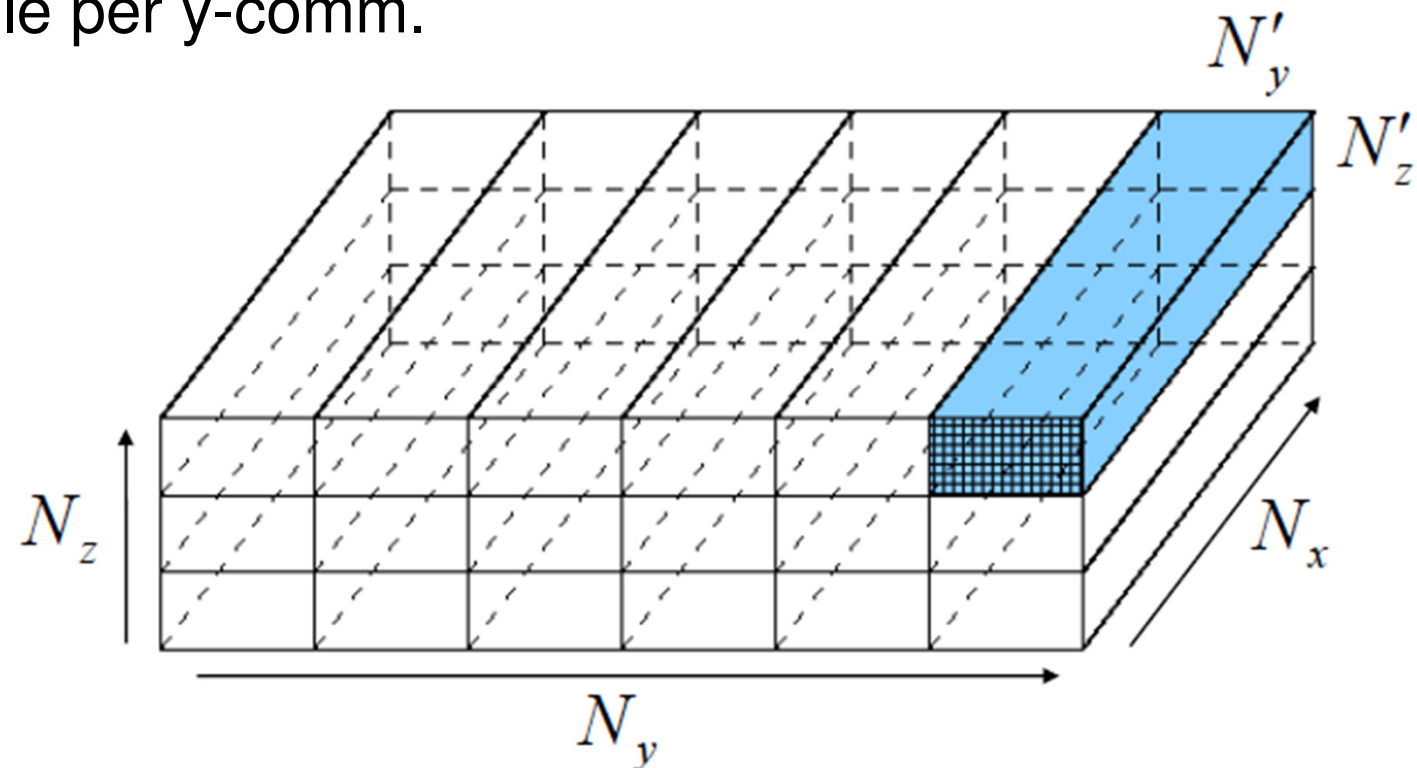
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_j u_i}{\partial x_j} = \frac{1}{\text{Re}} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial p}{\partial x_i} + \theta \delta_{i3}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial u_j \theta}{\partial x_j} = \frac{1}{\text{Re Pr}} \frac{\partial^2 \theta}{\partial x_j^2}$$

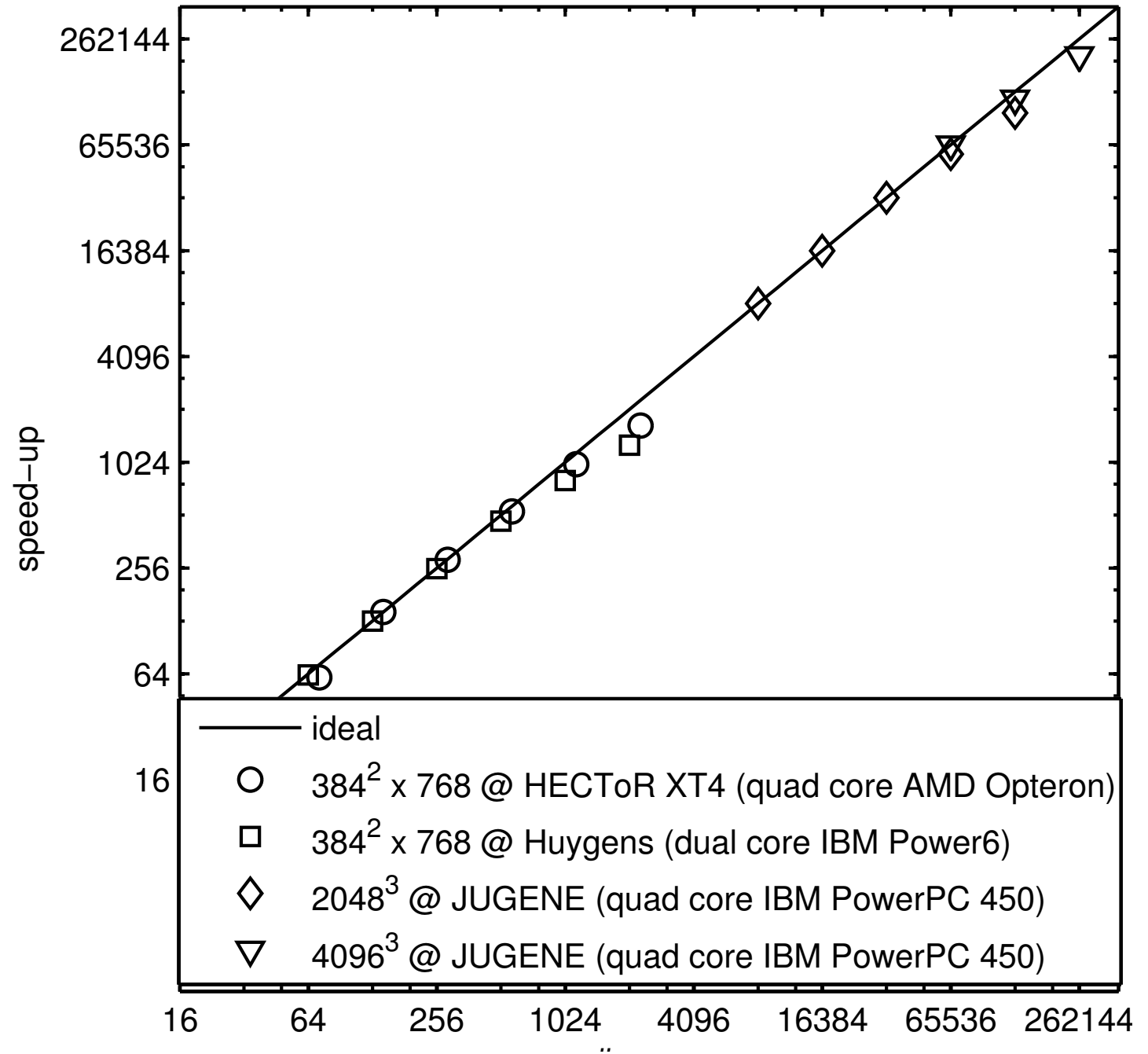
- (Navier-Stokes in Boussinesq approximation)
- Does not rely on empirical models
- Has become capable of reaching realistic Reynolds numbers on Tier-0 resources

DNS code sparkle

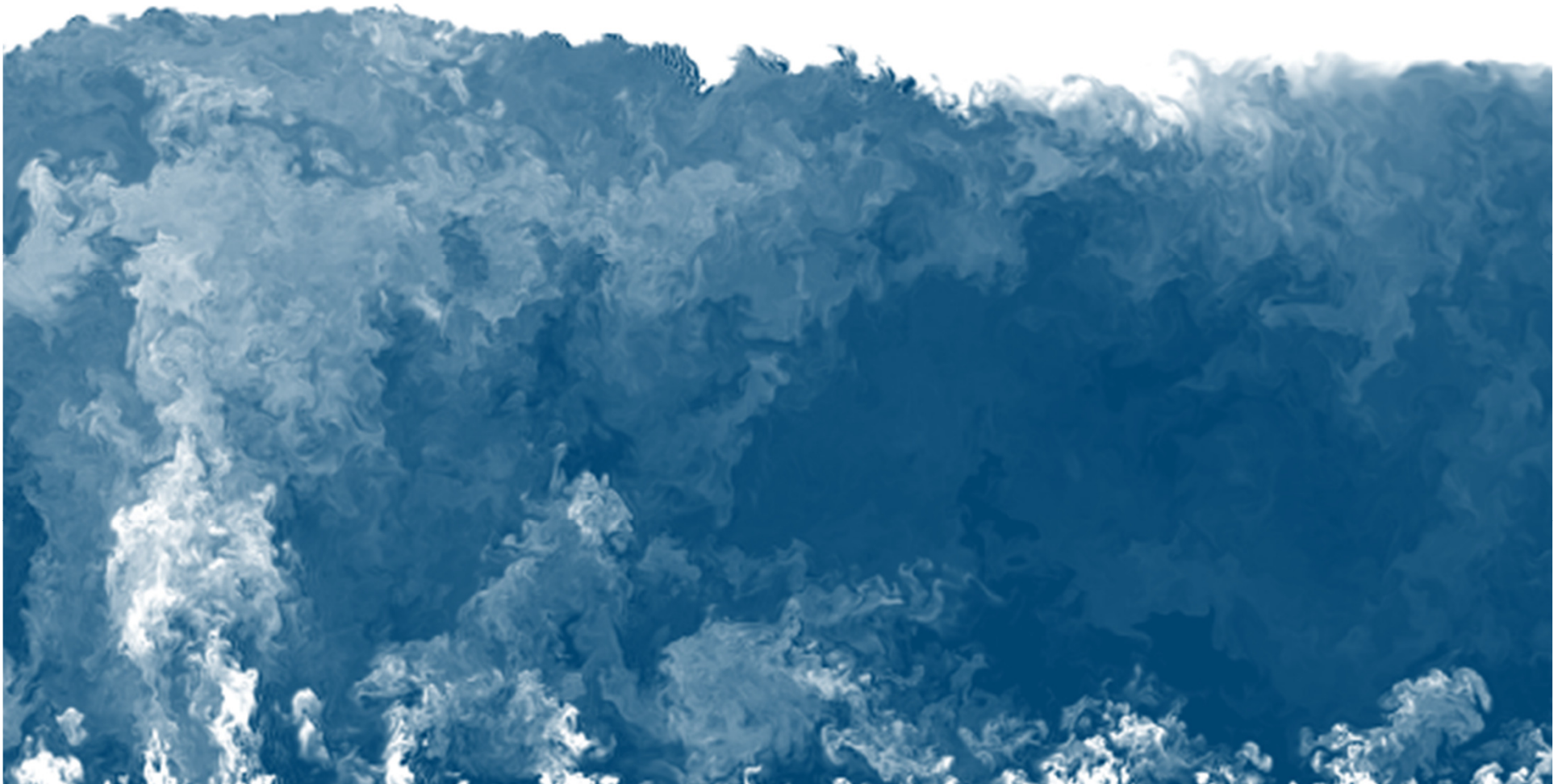
- Navier-Stokes equations in Boussinesq approximation;
- Fortran 90 with MPI and FFTW libraries;
- 2D domain decomposition using pencils;
- IO: one file per y-comm.

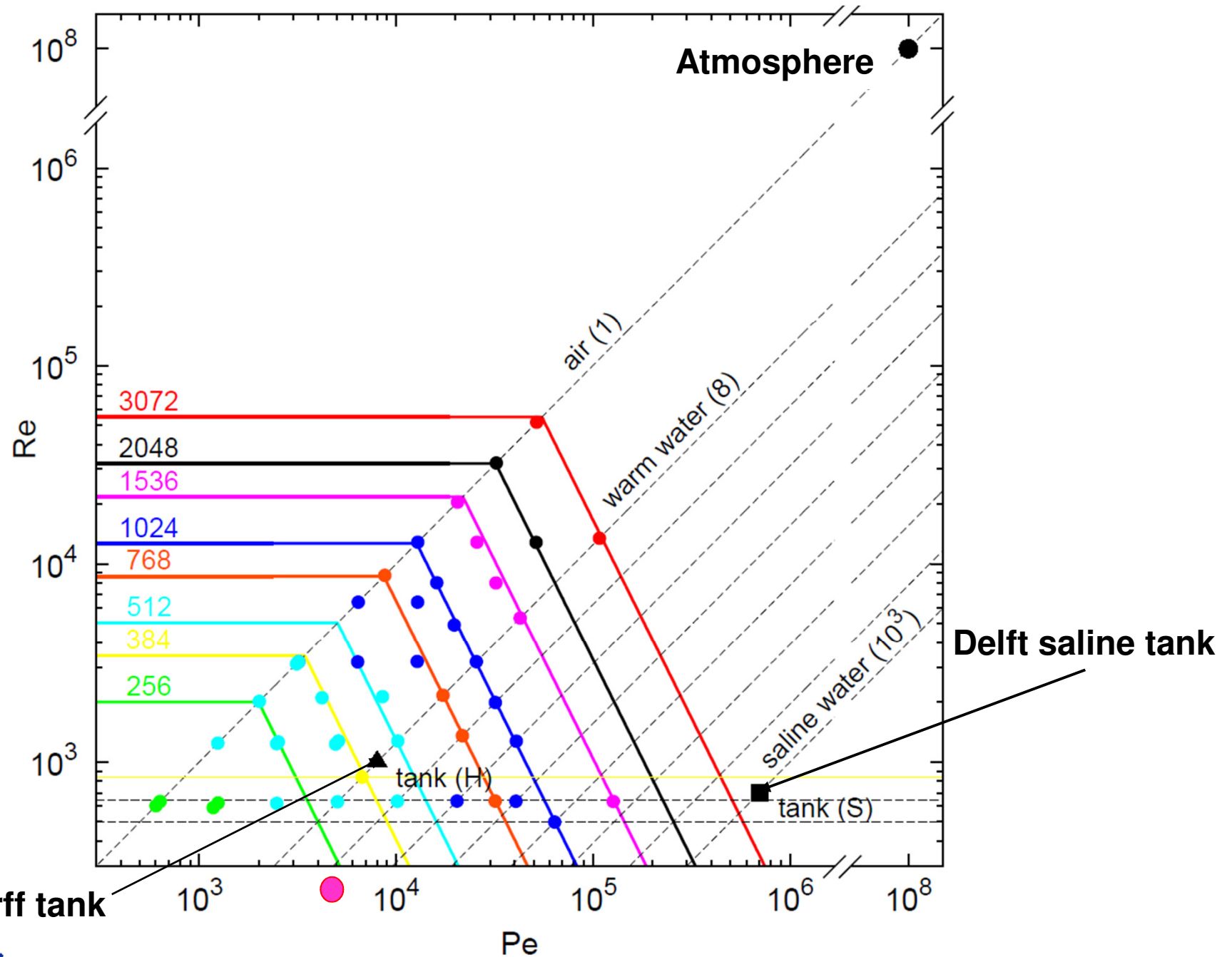


Speed-up



Homogeneous entrainment





Simulations performed

Site	Architecture	Max nr of cores	Grid
SARA	IBM Power 6	1024	1024 x 1024 x 768
CINECA	IBM BCX/5120	2048	2048 x 2048 x 1024
LRZ	SGI Altix 4700	3072	1536 x 1536 x 768
Juelich	Bluegene	32,768	3072 x 3072 x 1536

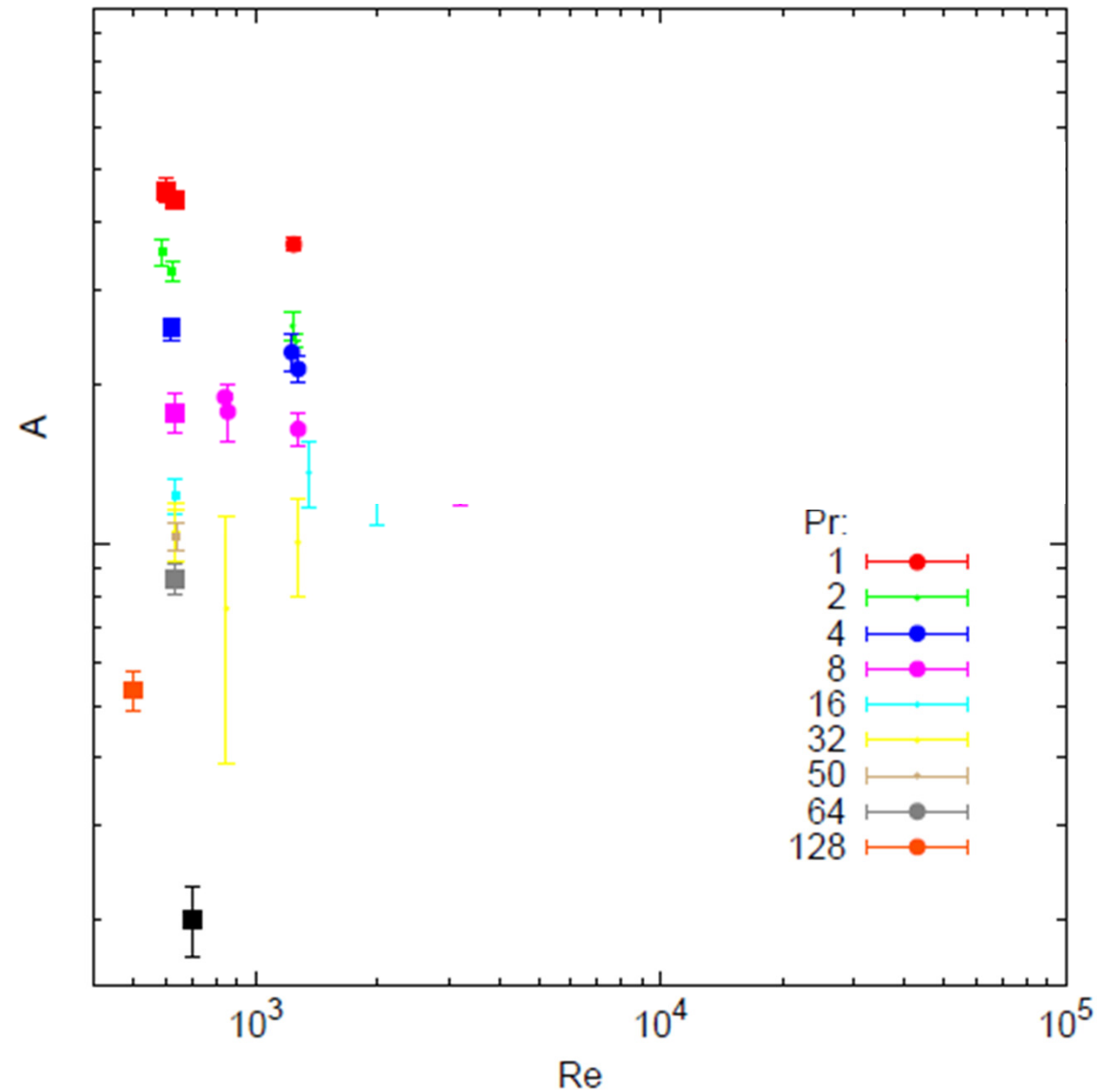


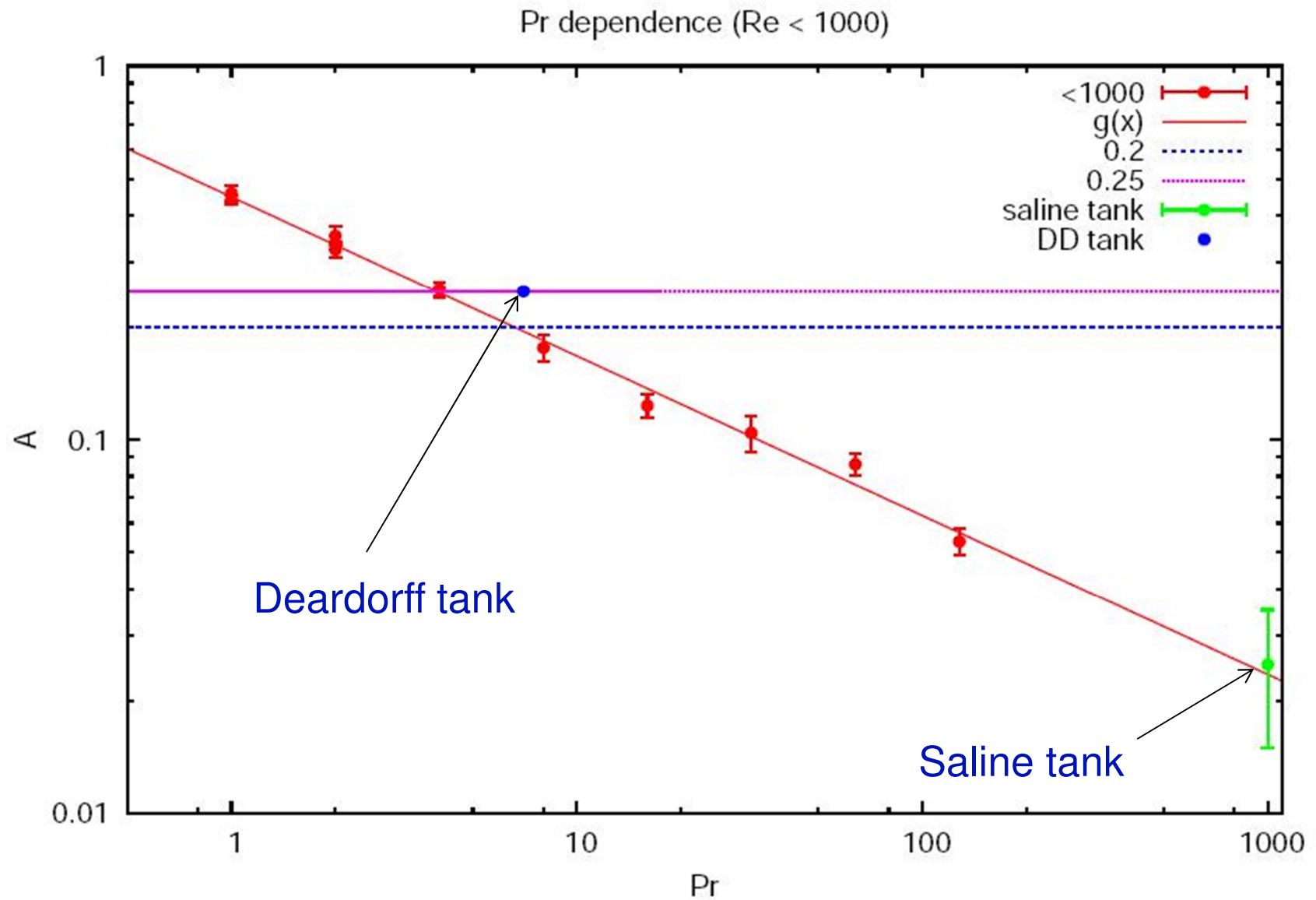
3Mhr



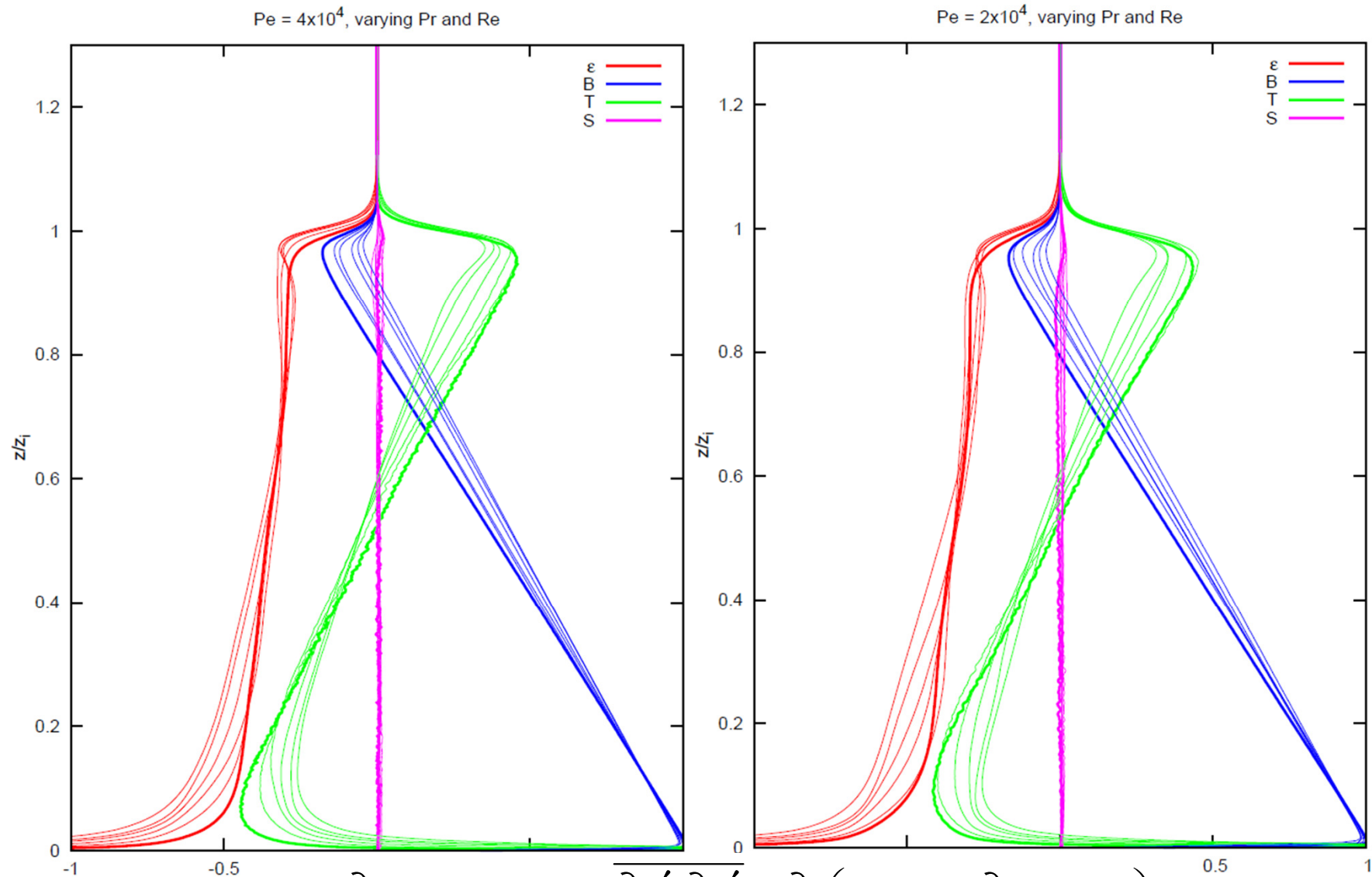
35Mhr

The need for extreme computing

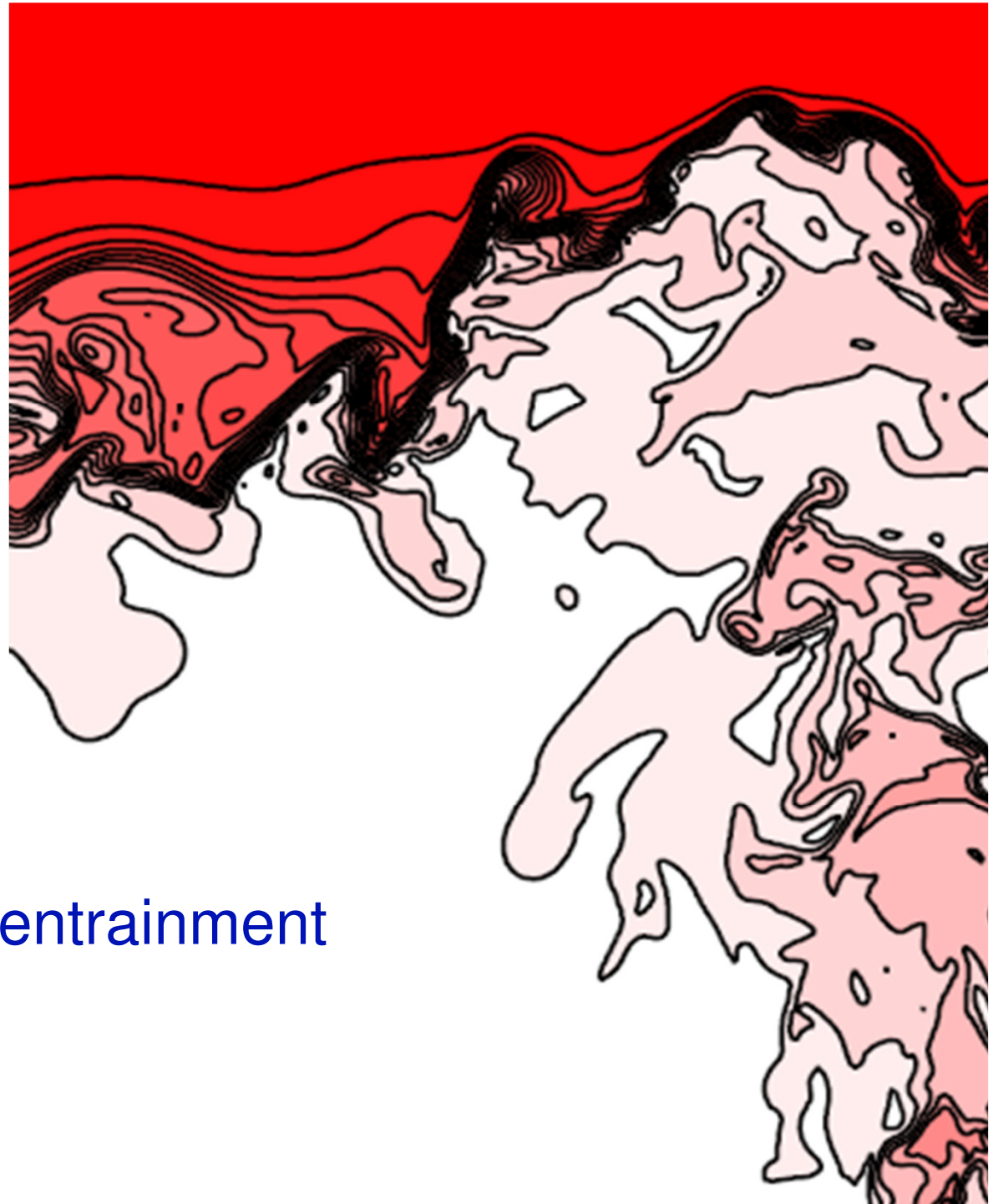




Asymptotic regime NOT reached in laboratory experiments



$$\underbrace{\frac{\partial e}{\partial t}}_S = \underbrace{\beta g \overline{w' \theta'}}_B - \underbrace{\nu \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}}_{\varepsilon} - \underbrace{\frac{\partial}{\partial z} \left(\overline{w' e'} - \nu \frac{\partial e}{\partial z} + \overline{w' p'} \right)}_T$$



Localised entrainment

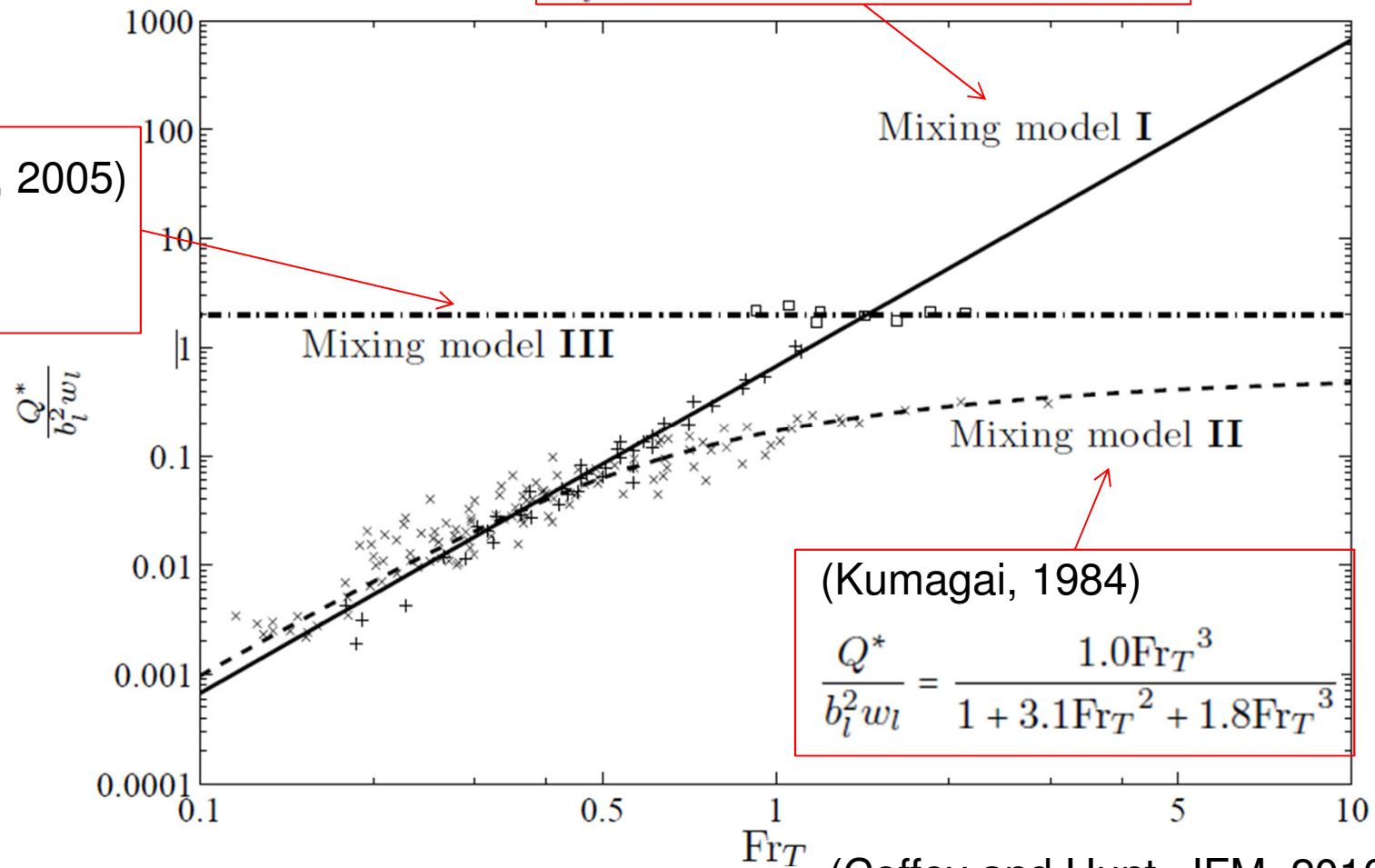
Interfacial mixing

(Baines, 1975)

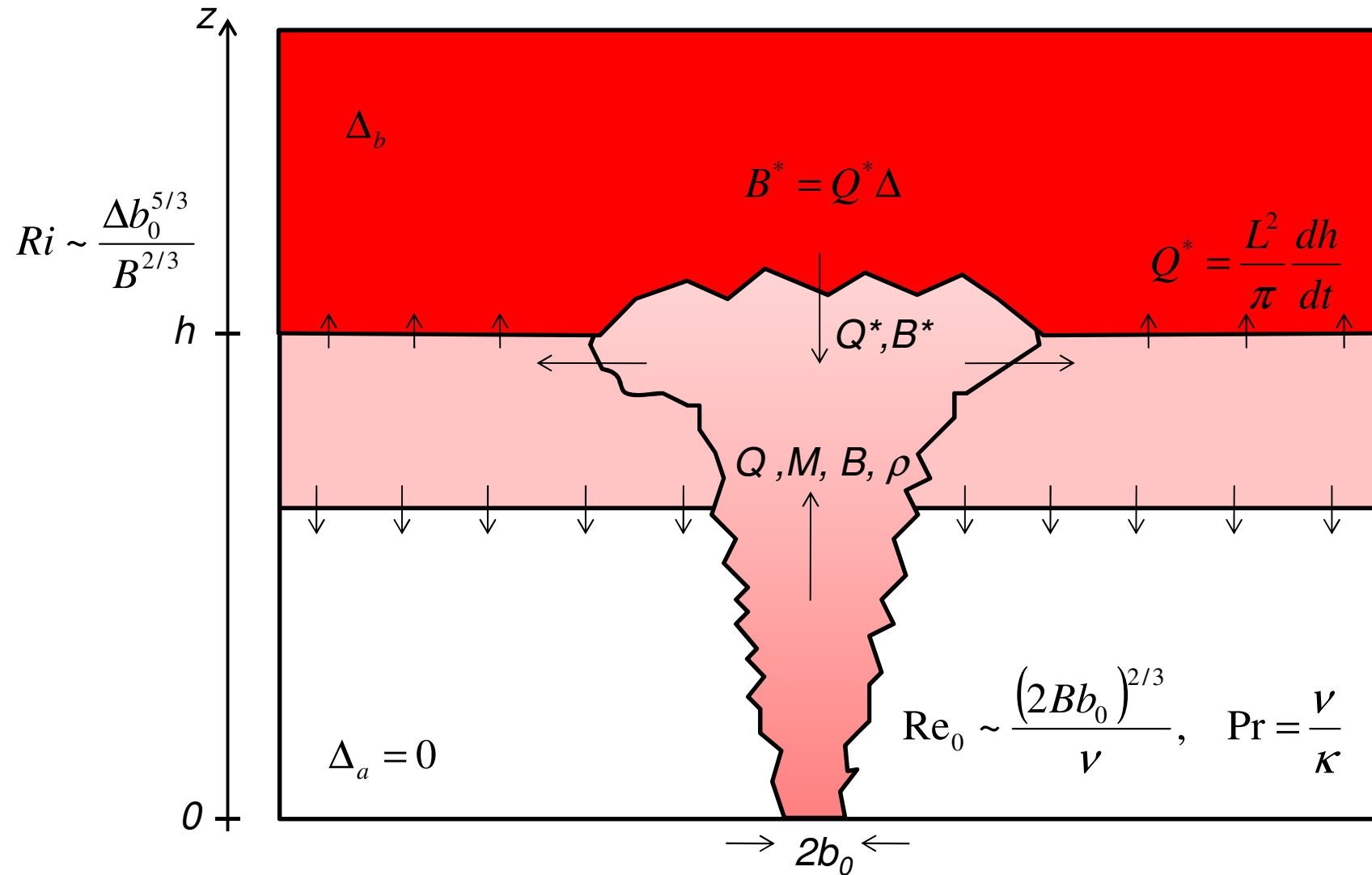
$$\frac{Q^*}{b_l^2 w_l} = 0.67 \text{Fr}_T^3, \quad \text{Fr}_T = \frac{w_l}{\sqrt{b_l \Delta g'}}$$

(Lin & Linden, 2005)

$$\frac{Q^*}{b_l^2 w_l} \approx 2.0.$$



Entrainment across density interface

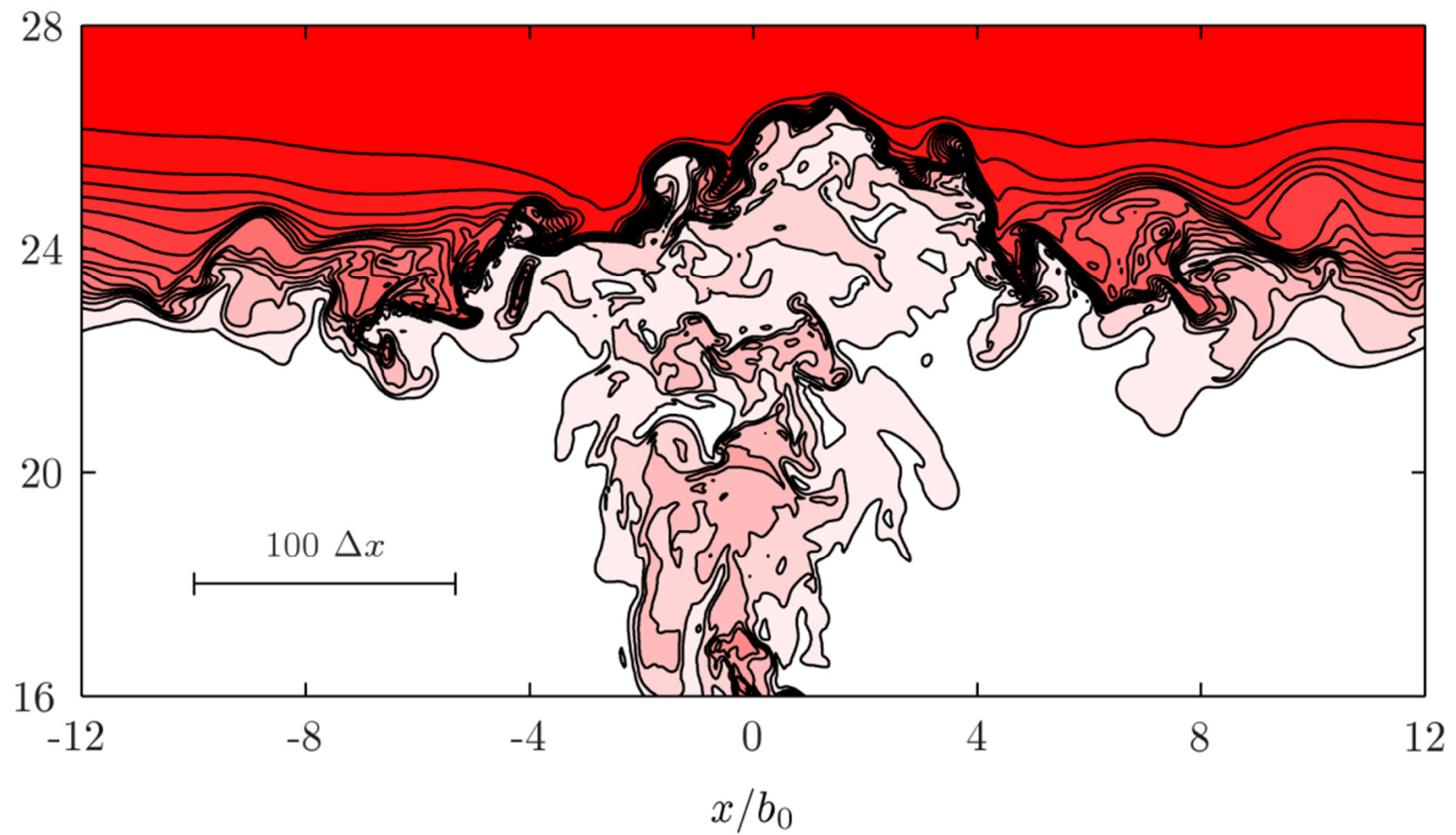


Simulation details

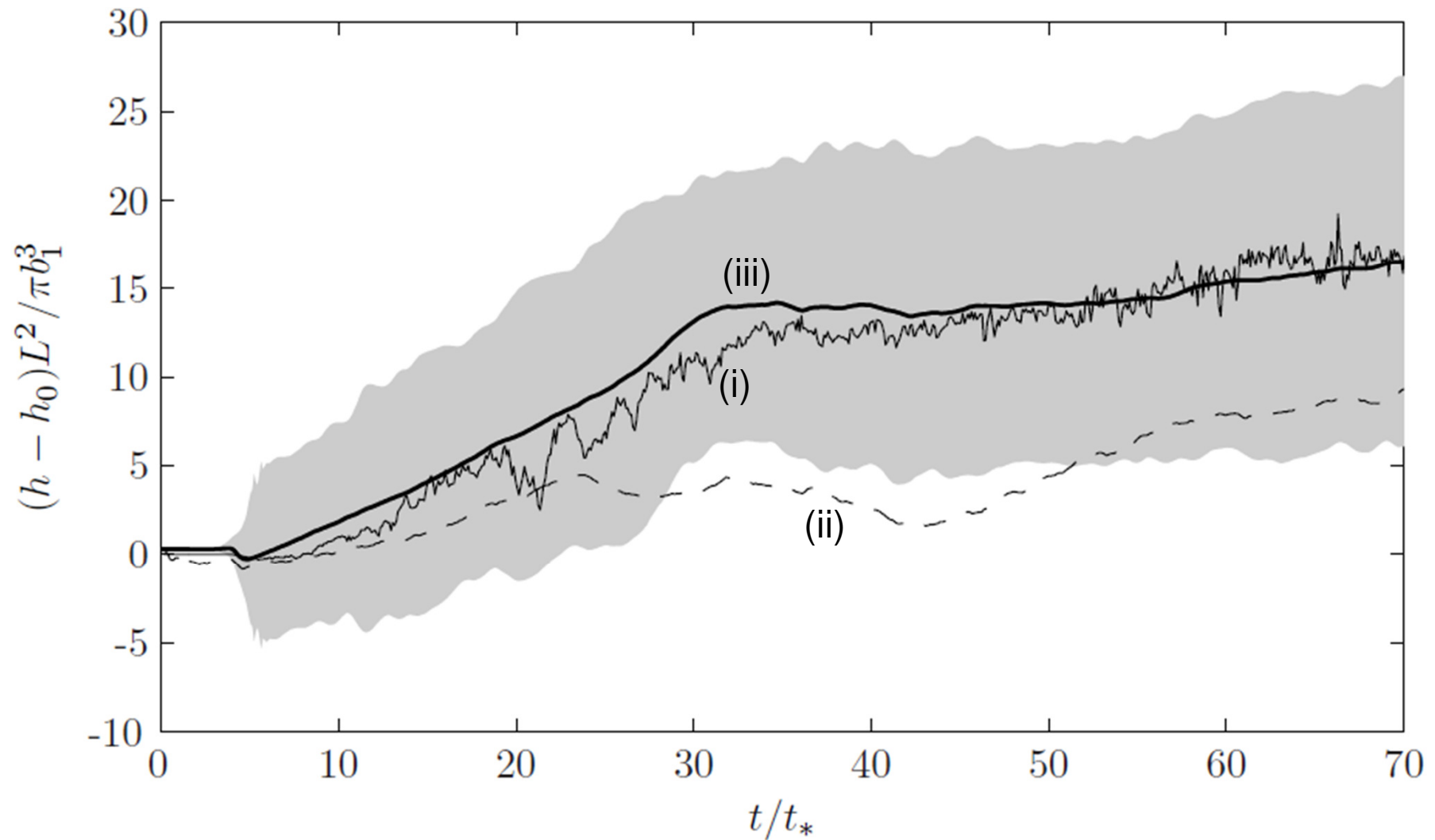
Re_0	Pr	Ri_0	Resolution	#procs	Required resources [CPU-hr]
600	0.71	various	1024x1024x768	8,192	200,000
1000	0.71	various	1536x1536x1024	32,768	600,000
1500	0.71	various	2048x2048x2048	65,536	3,000,000



30 Mhrs

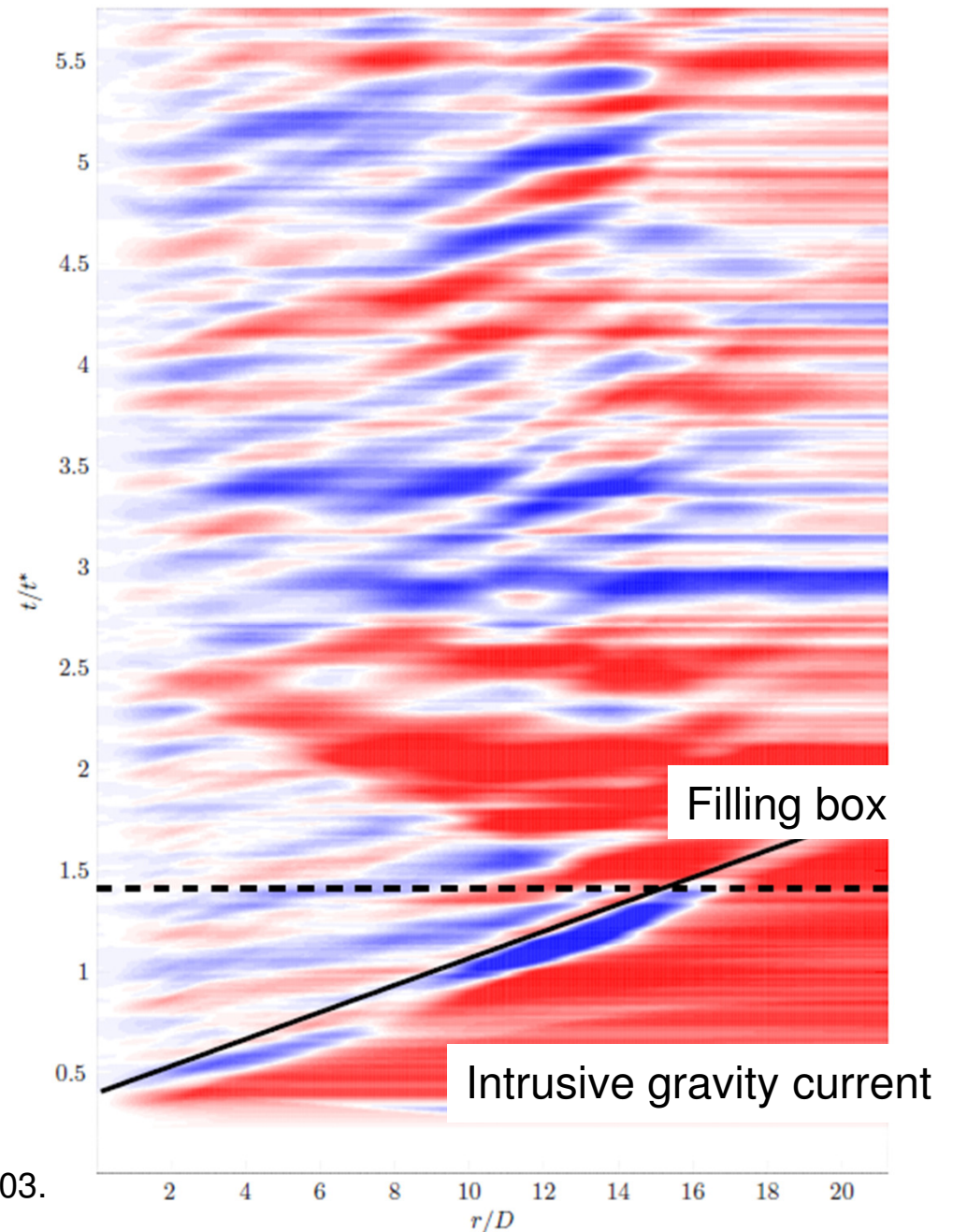


Evolution of interface position over time



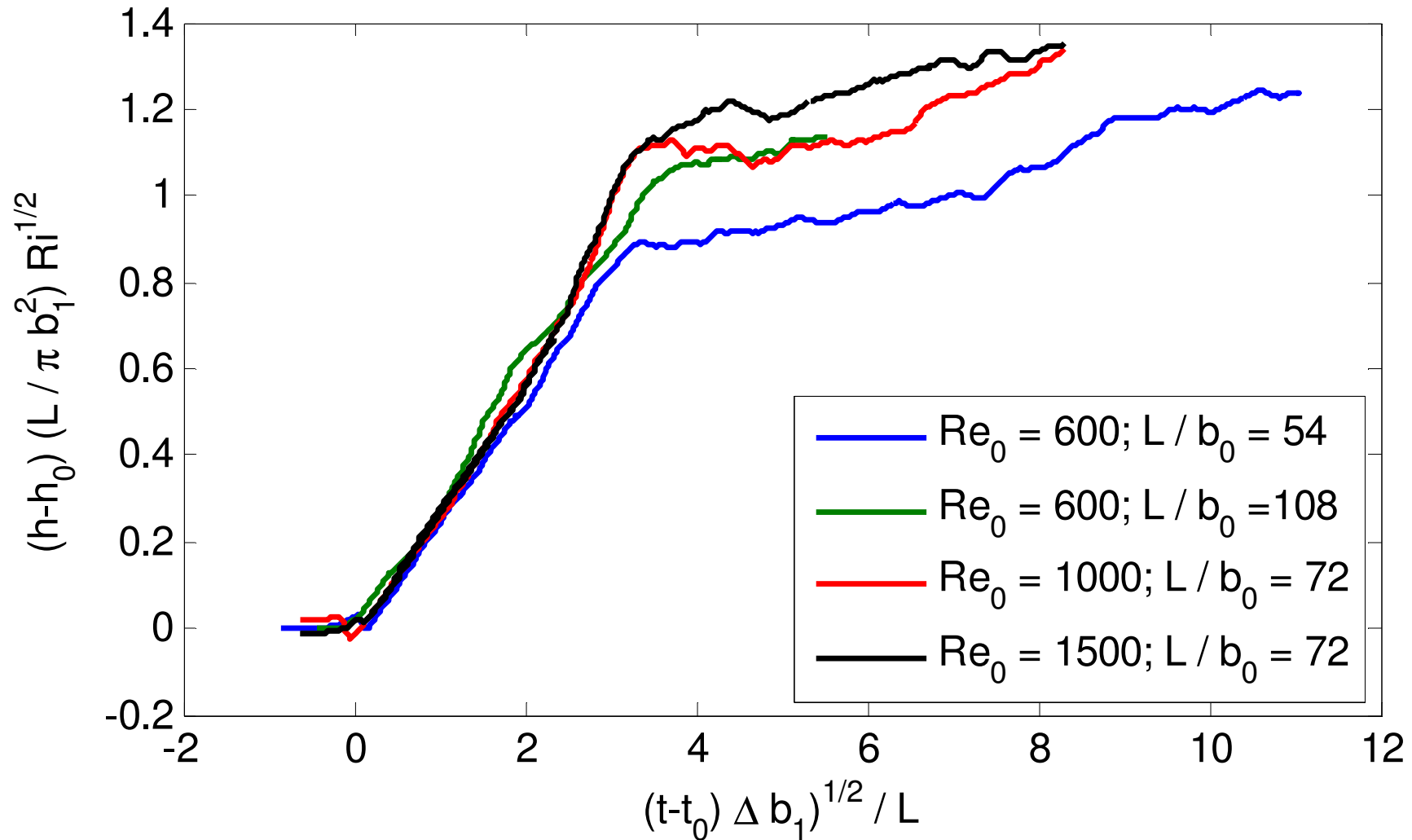
Why the kink?

- Associated with transition from intrusive gravity current to filling box behaviour
- Front velocity consistent with predictions for intrusive gravity current (e.g. Holdsworth, 2012).



HOLDSWORTH *et al.* (2012) *Phys. Fluids* **24**, 036603.

Interface positions for various L and Re



Dependence on Richardson number (ongoing)

sim 480



sim 469



sim 455



sim 478



Conclusions

Homogeneous entrainment:

- In the asymptotic regime ($Pe, Re \rightarrow \infty$) the appropriate law is indeed

$$w_e = w_* \frac{A}{Ri}$$

- Prefactor still uncertain; best estimate remains $A = 0.25$
- Laboratory experiments on entrainment have been influenced too much by molecular processes (Re, Pr)
- Improved parameterisations can be obtained by studying TKE budgets (ongoing)

Localised entrainment

- A plume in a filling box has a different entrainment behaviour than a plume in an unbounded domain;
- Work in development; much of the data still needs to be processed.

A bright future with DNS ideal experiments